Nearly all of what you need to know about quantum scattering theory for atom-interferometers and atomic clocks

Scattering theory
Threshold behaviors
Cold collision shifts, trap state changes (thermalization),
mean field,
Identical particle scattering

What you need to know about quantum scattering theory for atom-interferometers and atomic clocks

Everything we know is from scattering.

Scattering is different than most quantum problems.

No interaction, interact, then go to infinity with no interaction

When there is no scattering, the outgoing wavefunction still exists – it's just the initial wavefunction. Scattering amplitude is all of the effect of the scattering.

We often, usually in atomic physics, have central potentials V(r), independent of orientation.

Sakurai, *Modern QM* (concise) Gottfried, *QM Vol 1: Fundamentals*

Partial wave expansion

Initial wave function has to be a wave packet. It has a finite extent.

1. Expand in a basis of momentum plane waves.

2. Then expand plane waves in a basis of spherical waves
   - Central potential
   - Only one spherical wave, s-wave, at ultracold temperatures

- $j_\ell(kr)$ is a spherical standing wave – incoming and outgoing spherical waves.

- Partial wave ℓ scattering amplitude depends on energy (k)

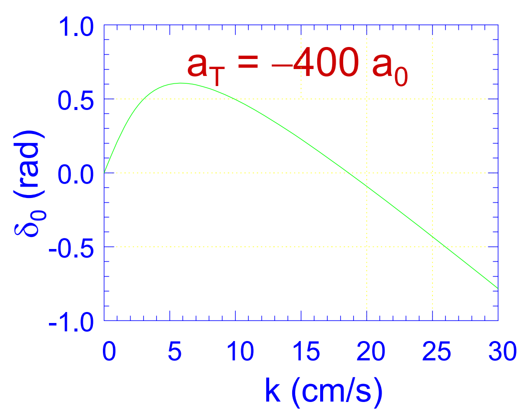
3. For elastic scattering, the scatterer can only shift the phase of a partial wave. It cannot change the amplitude.

Scattering phase shifts and s-wave scattering lengths

S matrix: $S_\ell = \exp(2i\delta_\ell)$
Scattering amplitude is:

ℓ=0 is s-wave, no angular momentum
δ_0= -k a, a is scattering length

Solve Schrödinger eq. to get δ_ℓ.



Every bound state give π phase shift to δ_0. 0 phase shift for infinite k.
$\exp(2i\delta_\ell)$ works the same for "scattering" a laser off an optical cavity.
Phase shift at k=0 is # of bound states + a fraction
- Scattering resonance when bound state at E=0.

At low k (energy), deBroglie wavelength is much longer than the range of V(r).
Scattering length is what is "looks" like far away.

Angular momentum barrier

For low energy, angular momentum barrier excludes sampling V(r).

- Only s-wave scattering at ultracold temperatures.

Only s-wave scattering when range of V(r) is shorter than deBroglie wavelength.

- Isotropic scattering – $d\sigma/d\Omega$=1, just like billiard balls.
- Cs deBroglie wavelength is 300 nm at 1μK.

The impact parameter required to have an angular momentum of hbar is so big that V(r) is 0, so no scattering.

s-waves probe entire V(r).

p-waves probe next zone, etc.

KG, Chang, Legere, PRL '95.

Differential and Total Cross sections

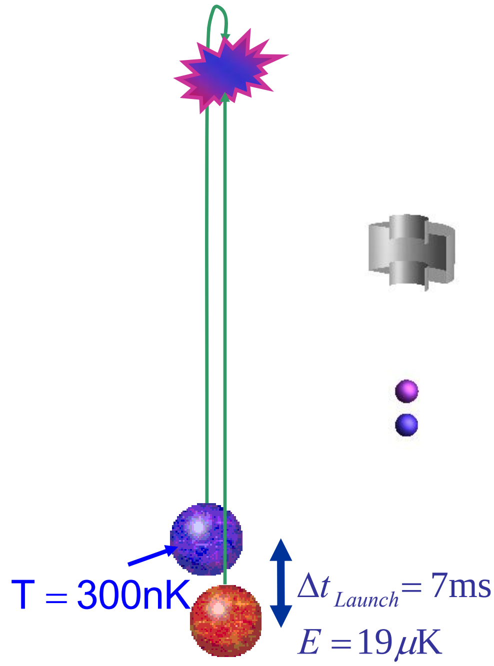
Differential cross section is the square of the scattering amplitude.

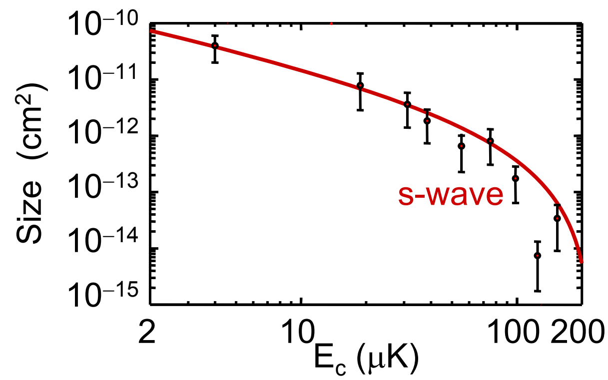
Integrate over all angles and use orthogonality of P_ℓ's to get total cross section.

What happens as phase shift goes through multiple of π as k increases?

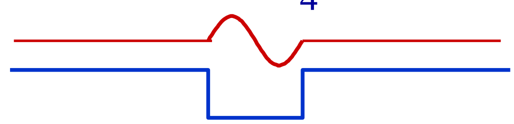
KG, Chang, Legere, PRL '95.

Ramsauer-Townsend effect





$$E_c = \frac{mg^2\Delta t^2}{4}$$



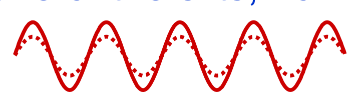
- State-to-state velocity-selected differential crossed-beam scattering at μK energies.

No scattering when phase shift is a multiple of π.

Legere & KG, PRL ('98)

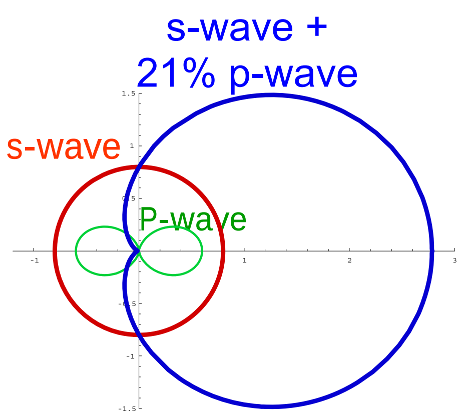
Young's 2 Slit Experiment

If you cut the intensity of the light by ½ through one of the slits, how much does the fringe contrast go down by?

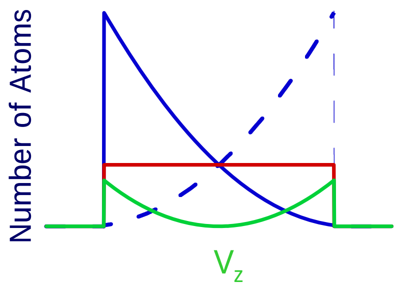


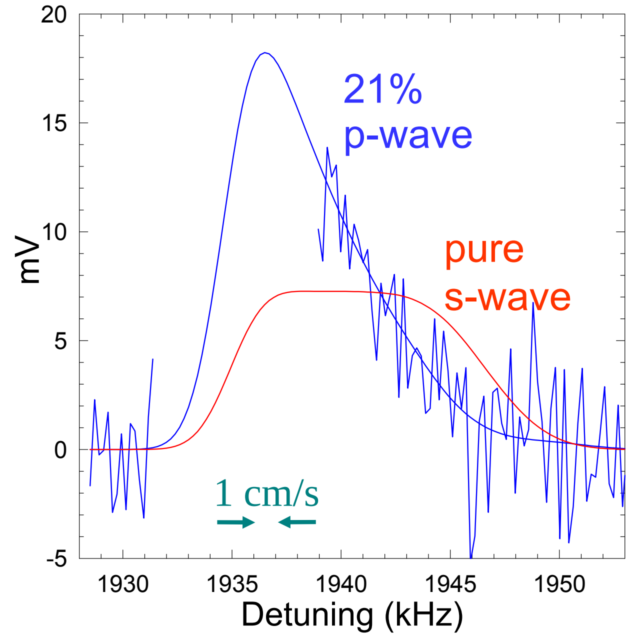
If your p-wave cross section is 10% of the s-wave, does it change the scattering much?

Quantum Interference of s & p-waves



Interference Term Dominates





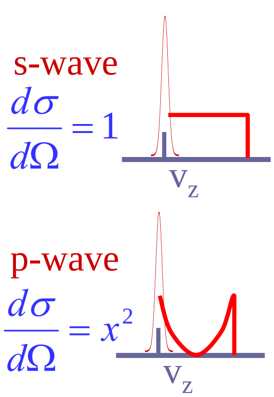
launch delay of 9 ms

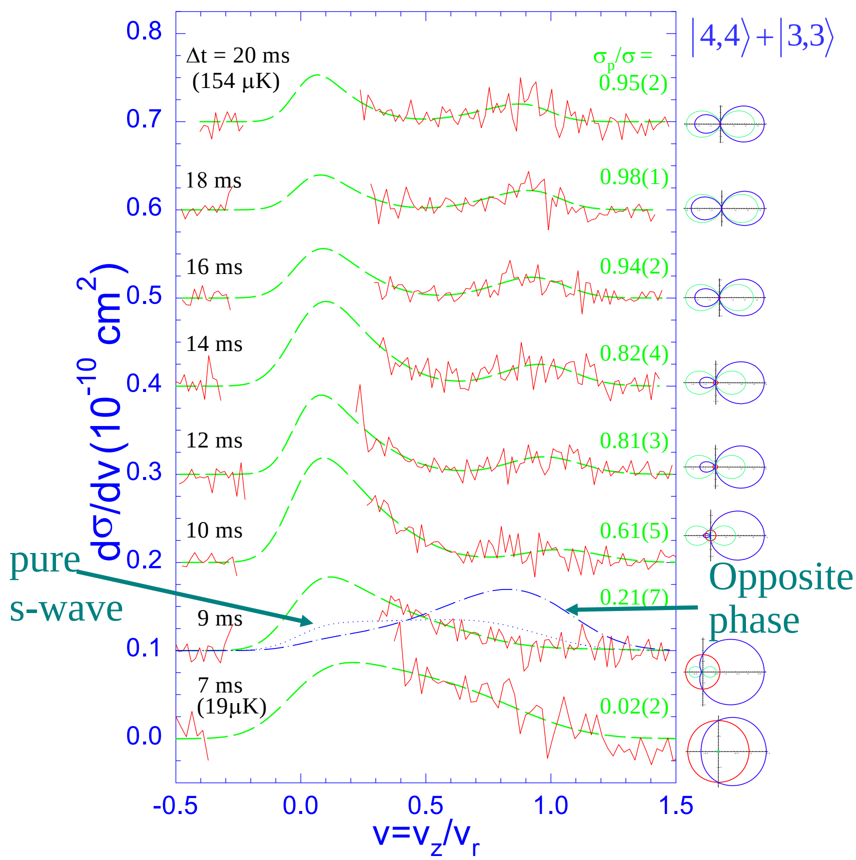
$$4\pi \frac{d\sigma}{d\Omega} = \sigma_s \pm 2\sqrt{3\sigma_s\sigma_p}\cos(\Theta) + 3\sigma_p\cos^2(\Theta)$$

$$\sigma = \sigma_s + \sigma_p + \cdots$$

KG, Chang, Legere, PRL '95

Quantum Interference of s & p-waves





Identical particles

What changes if the two colliding particles are identical?
Boson total wavefunction must be symmetric and fermions antisymmetric under exchange of particle labels.

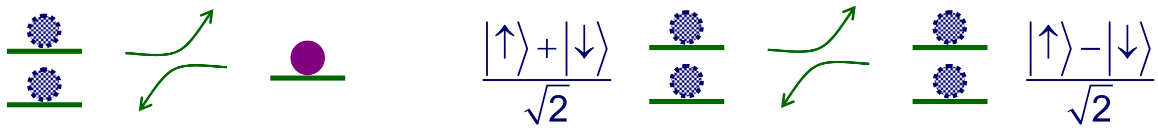
For particles in the same internal & spin states:
Bosons: only **even** ℓ are allowed - no p-wave scattering
Fermions: only **odd** ℓ are allowed - no (s-wave) scattering as $T \to 0$!

– For s-wave, two identical fermions can't be in the same place.

24Mg & 25Mg

What's the difference in their scattering lengths?

Scattering of coherent superposition(s)



One or both particles have internal structure – 2 levels that have a relative phase

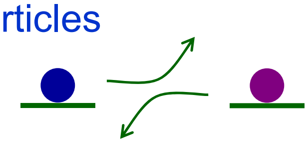
Both internal states scatter as before so outgoing wavefunction has an internal coherence.

1. Optical theorem - what happens in forward direction?
2. Density-dependent frequency shift in clocks and atom-interferometers
3. Symmetrization and non-identical coherences
4. Direct phase, collision shift vs. thermalizing collisions
5. Scattering in traps instead of free space.

Optical theorem

First, back to a single internal state, distinguishable particles

What happens in forward direction?



$$\Psi = \left[e^{ikz} + f(\theta)\frac{e^{ikr}}{r} \right]$$

Calculate probability current – measure flux of scattered atoms.

$$j = \hbar/\mu \,\mathrm{Im}\left(\langle \Psi | \vec{\nabla} | \Psi \rangle \right)$$

Three terms:

1. Unscattered wave function (only forward direction, θ=0)
2. Scattered current (all θ)

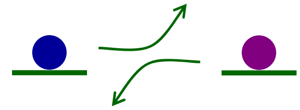
$$j(\theta) = \frac{\hbar k}{m}\cos(\theta) + \frac{\hbar k}{m}\frac{|f(\theta)|^2}{r^2} + j_{\mathrm{int}}(\theta = 0)$$

3. Interference current between unscattered and scattered (only forward direction, θ=0)

Gottfried '66, *QM Vol 1: Fundamentals* p. 107

Optical theorem & Interference current

$$\Psi = \left[e^{ikz} + f(\theta)\frac{e^{ikr}}{r} \right] \qquad j = \hbar/\mu \,\mathrm{Im}\left(\langle \Psi | \vec{\nabla} | \Psi \rangle\right)$$



Interference current

leading term with $e^{ikz} = e^{ikr\cos(\theta)}$:

$$j_{\mathrm{int}}(\theta) = \frac{\hbar k}{2mr}\left[1+\cos(\theta)\right]\left[e^{ikr[1-\cos(\theta)]}f(\theta) + c.c.\right]$$

$f(\theta)$ is smooth around $\theta=0$ so use $f(0)$. Integrate $j_{\mathrm{int}} r^2 d\Omega$ over 0 to small $\delta\theta$

$$j_{\mathrm{int}}(\theta) = i\frac{2\hbar\pi}{m}\left\{\left[1 - \tfrac{1}{2}e^{ikr[1-\cos(\delta\theta)]}\left(1+\cos(\delta\theta)\right)f(0) - c.c.\right]\right\}$$

Remember there is a spread in k so oscillating term averages to 0.

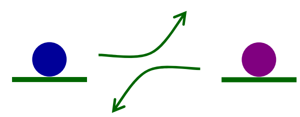
$$j_{\mathrm{int}}(\theta) = -4\pi\frac{\hbar}{m}\mathrm{Im}\left[f(0)\right]$$

Scattering in forward direction is dominated by j_{int} and represents loss of atoms.

Gottfried '66, *QM Vol 1: Fundamentals* p. 107

Optical theorem & Interference current

$$j_{\mathrm{int}}(\theta) = -4\pi\frac{\hbar}{m}\mathrm{Im}\left[f(0)\right]$$



Using definition of $f(\theta)$, or just consider flux conservation,

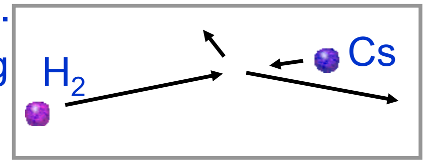
$$\mathrm{Im}\left[f(0)\right] = \frac{4\pi}{k}\sigma_{tot}$$

There are phase shifts in the forward direction. Following this same, perhaps familiar, procedure gives you frequency shifts of coherence (cold collision shift in clocks).

Gottfried '66, *QM Vol 1: Fundamentals* p. 107

Scattering of Cold Atom Coherences by Hot Atoms: Background Gas Collision Shifts

- Current estimates of ~10^{-16} come from measurements of clock shifts of room temperature Cs atoms.
- Large cross section for small angle scattering
  - Negative frequency shift
- Hard-core, large angle scattering
  - Positive frequency shift
- They partially cancel each other.



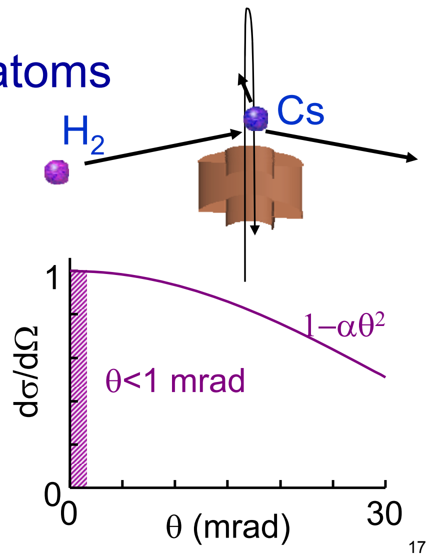
Beer & Bernheim, PRA '76

Room-temperature collisions with cold atoms

- Atoms are not detected if $\Delta v > 3$cm/s.

$$\lambda_{dB} = \frac{h}{mv} = 100nm$$

- If $\Delta v < 3$cm/s, you cannot localize the atom to $\Delta b < 100$nm.
- $\Rightarrow$ Quantum/Diffractive Scattering
  - ~1% of the diffractive cone – negligible
  - Only forward scattering contributes



KG, PRL '13

Scattering of Coherent Superpositions

- Cold clock atom is in a coherent superposition of states 1 & 2.
- Consider each state scatters off a hot background gas atom.

$$\left|\Psi\right\rangle = \frac{\left|1\right\rangle + \left|2\right\rangle}{\sqrt{2}} e^{ikz} \quad \Rightarrow \frac{1}{\sqrt{2}}\left[e^{ikz} + \frac{e^{ikr}}{r} f_1(\theta)\right]\left|1\right\rangle + \frac{1}{\sqrt{2}}\left[e^{ikz} + \frac{e^{ikr}}{r} f_2(\theta)\right]\left|2\right\rangle$$

- 2nd $\pi/2$ pulse yields a Ramsey fringe.
- Probability current of state 2 has 3 terms: unscattered, interference, scattered.

$$j = \frac{\hbar}{\mu}\operatorname{Im}\left(\left\langle\Psi\middle|2\right\rangle \nabla \left\langle 2\middle|\Psi\right\rangle\right) = \frac{\hbar k}{\mu}\cos^2\left(\frac{\phi}{2}\right)\cos(\theta) + j_{2,\text{int}}(\theta=0) + \cancel{j_{2,sc}(\theta)}$$

$$j_{2,\text{int}} = \frac{\pi n \hbar}{\mu}\left\{\operatorname{Re}\left[f_1(0) - f_2(0)\right]\sin(\phi) - 2\operatorname{Im}\left[f_1(0) + f_2(0)\right]\cos^2\left(\frac{\phi}{2}\right)\right\}$$

$\theta=0$ dominates for ultracold atoms.

with

$$f_1(\theta) = \sum_{\ell=0,\infty}(2\ell+1)\frac{e^{i\delta_{1\ell}}\sin(\delta_{1\ell})}{k}P_\ell\left[\cos(\theta)\right]$$

$$j_{2,\text{int}} = \frac{\pi n \hbar}{2\mu k}\sum_\ell (2\ell+1)\Big(\left[\sin(2\delta_{1\ell}) - \sin(2\delta_{2\ell})\right]\sin(\phi) - 4\left[\sin^2(\delta_{1\ell}) + \sin^2(\delta_{2\ell})\right]\cos^2\left(\frac{\phi}{2}\right)\Big)$$

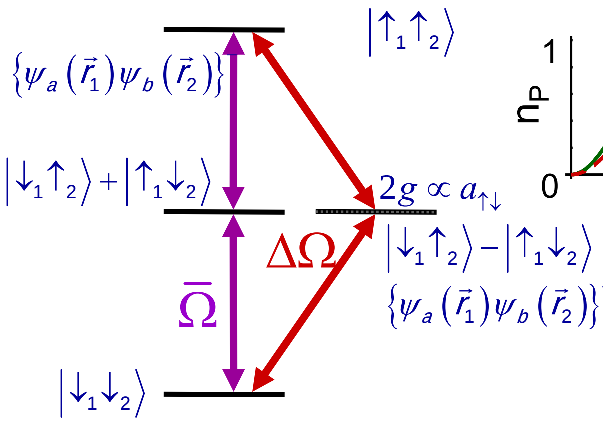
frequency shift **amplitude**

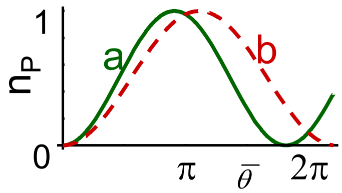
KG, PRL '13

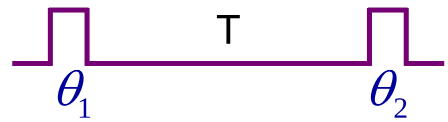
Scattering of 2 coherent superpositions

- Many particles are sum of pair-wise effects
- Basis – Singlet and Triplet states of 2 atoms:

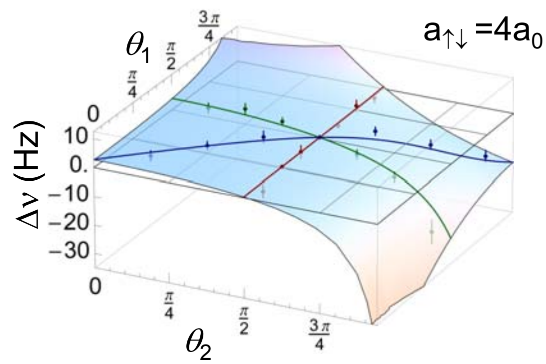
$\Omega = \Omega_a e^{i\Delta\omega_a t}$ $\Omega' = \Omega_b e^{i\Delta\omega_b t}$







$$\Delta\nu = \frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$



- $\Delta\theta_1=0 \Rightarrow$ s=0; identical fermions
  - $\Delta\nu$ is not proportional to $n_S - n_P$

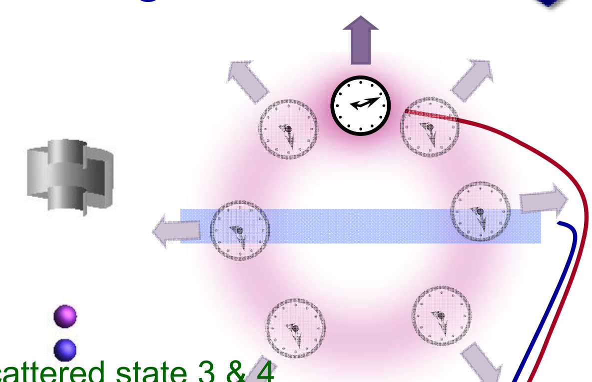
$$\cos(\bar{\theta}_1) = \frac{n_S - n_P}{n}$$

- No shift for $\Delta\theta_2=0$
- No shift for $\bar{\theta}_2 = \pi/2$

KG, PRL '09
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

Precise Measurement of Scattering Phase Shifts

- Juggle atoms by tossing 2 laser-cooled clouds with short delay.
- Launch delays of 7 to 20 ms give ultra-cold scattering, 15 to 200μK.
- In a clock, a microwave cavity prepares atoms in a coherent superposition and enables a readout of the relative phase of those two clock states.

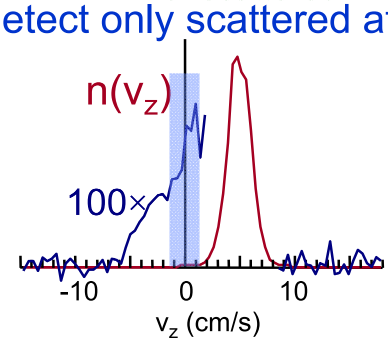


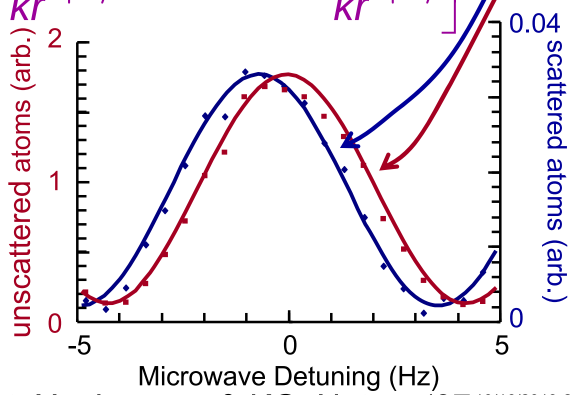
scattered state 3 & 4

$$\psi^+ = \frac{1}{\sqrt{2}}\left[e^{+ikz}\left(|3\rangle+|4\rangle\right) + e^{i\delta_3}\sin\delta_3\frac{e^{ikr}}{kr}|3\rangle + e^{i\delta_4}\sin\delta_4\frac{e^{ikr}}{kr}|4\rangle\right]$$

unscattered

- Detect only scattered atoms.





Hart, Xu, Legere, & KG, Nature '07