Precision Measurements of Scattering Phase Shifts in a Fountain Clock

- How can we measure differential cross-sections – s-wave or p-wave?
- Juggling fountain
- Connection between scattering and frequency shifts.
- Measuring quantum scattering phase shifts with clock-like accuracy – time variation of me/mp?
- Frequency shifts from background gas collisions

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Probing s-wave Scattering

1. Select atoms with narrow distribution of v_z.

2. Elastic collisions change v_z.


4. Scan probe over velocity distribution.

5. Detect with F=4-5’ fluorescence

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**KG, Chang, Legere PRL ’95**
Direct observation of s-wave scattering

- $T=0.9\mu K$ for collisions.
- 7% of atoms collide
- Early clearing gives no collisions
- Late-Early = collisions

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Direct observation of s-wave scattering

- Single parameter fit to data set gives $\sigma = 0.4 \times 10^6 \text{ Å}^2$
- 99.9(1)% s-wave at 0.9 $\mu K$
- (incoherent s&p give 97(3)% s wave)
- $d\sigma/d\Omega$ is highly sensitive to interference terms.
Measuring $d\sigma/d\Omega$

Detect scattering angle $\theta$ by measuring scattered velocity distribution.

Initial velocity distribution

Use Doppler shift on Raman transition to measure $n(v_z)$.

$\nu_z = \frac{1}{2} \nu_r \cos \theta$  

$P(\theta) \rightarrow P(v_z): \frac{dv_z}{d\nu} = \frac{1}{2} \nu_r d(\cos \theta) \propto d\Omega$

So

$P(v_z) \propto P(x = \cos(\theta))$

$= \left| \sum (2\ell + 1)e^{i\delta_i} \sin \delta_i P_i(\cos(\theta)) \right|^2$

$\frac{d\sigma}{d\Omega} = 1$

$\frac{d\sigma}{d\Omega} = v_z^2$

Juggling Atomic Fountain

1. Multiply load UHV MOT
2. Hold first cloud Trap and launch 2nd cloud from vapor cell MOT
3. launch first cloud, $T=1.5 \mu K$, hide in $F=3$
4. Trap 2nd cloud for 2 ms & launch
5. Optically pump both clouds to $|4,4\rangle$
6. Early/Late clearing of $|4,4\rangle$ atoms.

Velocity select 2nd cloud to $|3,3\rangle$

$\Delta v_z = \frac{2\Delta \nu}{\lambda} \sin(\theta)$

7. Scan velocity distribution-

8. Detect atoms.
Quantum Interference of s & p-waves

The figure shows the interference term dominates the number of atoms. The equation for the interference term is:

\[ \sigma = \sigma_s + \sigma_p + \cdots \]

where \( \sigma_s \) and \( \sigma_p \) represent the s-wave and p-wave amplitudes, respectively. The phase difference \( \cos(\delta_s - \delta_p) \approx 1 \) leads to a launch delay of 9 ms.

The figure also includes a plot of the number of atoms over detuning (kHz) and the corresponding mV output. The s-wave and p-wave contributions are highlighted, with the s-wave showing a more significant effect.

KG, Chang, Legere, PRL '95

Quantum Interference of s & p-waves

The figure illustrates the difference in d\sigma/d\Omega for s-wave and p-wave. The s-wave contribution is constant, while the p-wave contribution varies with \( x^2 \). The plot shows the change in d\sigma/dv (10^-10 cm^2) over different time delays (\( \Delta t \)). The pure s-wave and opposite phase are indicated with arrows.

The insert shows the change in \( \sigma_s/\sigma_p \) for different time delays, with values indicating the relative strength of the s-wave to p-wave interference.
S-wave energy dependence and p-wave quantum threshold

- Pure triplet
- $1/E$ ‘resonance’ behavior - constant phase, expected for $a < 0$
- $p$-wave $\propto E_c^2$
- $p$-wave: $a_p = -107(6)a_0$ - Mixture of singlet and triplet & singlet interactions.

A clock with 100x smaller collision shift?

- Select 1% of the velocity distribution in 1D.
- 2 clock $\pi/2$ pulses
- Probe the atoms that didn’t collide!
- Collision shift is interference in the forward direction between the unscattered and scattered amplitudes.
- In this clock, what is the phase shift of the scattered atoms?
Scattering of Coherent Superpositions

- Cold clock atom is in a coherent superposition of states 1 & 2.
- Each state scatters coherently.
  \[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left[ e^{ikz} + e^{ikr} f_1(\theta) \right] |1\rangle + \frac{1}{\sqrt{2}} \left[ e^{ikz} + e^{ikr} f_2(\theta) \right] |2\rangle \]
- 2\textsuperscript{nd} $\pi/2$ pulse with phase $\phi$ yields a Ramsey fringe.
  \[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left[ e^{ikz} + e^{ikr} f_1(\theta) + e^{-i\phi} \left( e^{ikz} + e^{ikr} f_1(\theta) \right) \right] |2\rangle \]
- Probability current of state 2: unscattered, interference, scattered.
  \[ j = \frac{\hbar}{\mu} \text{Im} \left( \langle \Psi | 2 \rangle \nabla \langle 2 | \Psi \rangle \right) = \frac{\hbar k}{\mu} \cos^2 \left( \frac{\phi}{2} \right) \cos(\theta) + j_{2,\text{int}}(\theta = 0) + j_{2,\text{sc}}(\theta) \]

For s-wave:

- Cold collision frequency shift
  \[ j_{2,\text{int}} = \frac{\pi \hbar}{2 \mu k} \left( \sin(2\delta_{1,0}) - \sin(2\delta_{2,0}) \right) \sin(\phi) - 4 \left[ \sin^2(\delta_{1,0}) + \sin^2(\delta_{2,0}) \right] \cos^2 \left( \frac{\phi}{2} \right) \]
- Scattered amplitude
  \[ j_{2,\text{sc}} = \frac{\pi \hbar}{\mu k} \left[ 4 \sin(\delta_{1,0}) \sin(\delta_{2,0}) \cos \left( \frac{\phi - (\delta_{1,0} - \delta_{2,0})}{2} \right) \right] + \left[ \sin(\delta_{1,0}) - \sin(\delta_{2,0}) \right]^2 \]

Precise Measurement of Scattering Phase Shifts

- Juggle atoms by tossing 2 laser-cooled clouds with short delay.
- Launch delays of 7 to 20 ms give ultra-cold scattering, 15 to 200\,\mu K.
- In a clock, a microwave cavity prepares atoms in a coherent superposition and enables a readout of the relative phase of those two clock states.
  \[ \psi^+ = \frac{1}{\sqrt{2}} \left[ \theta e^{ikz} |3\rangle + |4\rangle \right] + e^{i\delta_3} \frac{\sin \delta_3}{kr} |3\rangle + e^{i\delta_4} \sin \delta_4 e^{ikr} \frac{\sin \delta_4}{kr} |4\rangle \]
- Detect only scattered atoms.

\[ n(v_z) \]

\[ v_z \text{ (cm/s)} \]

\[ 10^3 \text{ arb.} \]

\[ \text{Microwave Detuning (Hz)} \]

Hart, Xu, Legere, & KG, Nature '07
Precise Measurement of Scattering Phase Shifts

- In scattering measurements, effects are proportional to atomic density.
- Best density measurements are 10%.
- Key is that the relative phase of clock coherence of the scattered atoms is independent of density.
- Clock-like accuracy: ppm scattering lengths.

Hart, Xu, Legere, & KG, Nature '07

A Quantum Scattering Interferometer

- Mach-Zehnder

- Phase shift, not frequency shift
- 8 mrad statistical error
- Now, 6 mrad in 20 min.
Direct Observation of the Variation of Scattering Phase Shifts through Feshbach Resonances

- Vary magnetic field to tune molecular bound states through resonance.
- Phase shift goes through a resonance.
- Energy spread of 800 nK ($E_c=32\mu K$) broadens resonance.
- Sensitive to time variation of $m_e/m_p$ at a Feshbach resonance.

![Graph showing phase shift variations](image)

Direct Observation of the Variation of Scattering Phase Shifts through Feshbach Resonances

- 4 different magnetic moments for each channel.
- Resolve multiple resonances.
- Isolate to 40,3m channel.

![Graph showing phase shift variations](image)
Feshbach Resonances & Time Variations

- Chin & Flambaum have shown sensitivity to time variation of $m_\phi/m_\rho$ of $10^9$ for mG wide Feshbach resonances.
- 100 $\mu$rad precision can give scattering lengths to 1 ppm.
- Paris group has observed frequency shifts from several narrow, low-field Feshbach resonances.
- Scattering phase shifts vary by $\pm\pi/2$ at resonances.

Chin & Flambaum, PRL ’06
Kokkelmans, Ph.D. theis, ’00
Marion, Bize, …, Kokkelmans, & Salomon, arXiv:physics/0407064 ’06

Quadratic Zeeman Shifts

- Magnetic field gradients shift clock frequency as $2 \times 427 \frac{Hz}{G} B_0 dB$
- Shift of transition is 106 Hz or 80 rad @ 0.5 G.
- No Feshbachs for $|3-3\rangle$
- Use $|3-3\rangle$ to measure gradients – scattered clock atoms follow the same trajectories.
Scattering of Cold Atom Coherences by Hot Atoms: Background Gas Collision Shifts

- Current fountain clock uncertainties of \(10^{-16}\) come from shifts of room temperature clocks.
- Large cross section for small angle scattering
  - Negative frequency shift
- Hard-core, large angle scattering
  - Positive frequency shift
- They partially cancel each other.

Room-temperature collisions with cold atoms

- Atoms are not detected if \(\Delta v>3\text{cm/s}\).
  \[
  \lambda_{db} = \frac{\hbar}{m v} = 100\text{nm}
  \]
- If \(\Delta v<3\text{cm/s}\), you cannot localize the atom to \(\Delta b<100\text{nm}\).
- \(\Rightarrow\) Quantum/Diffractive Scattering
  - \(~1\%\) of the diffractive cone – negligible
  - Only forward scattering contributes

Scattering of Coherent Superpositions

- Cold clock atom is in a coherent superposition of states 1 & 2.
- Each state scatters off a background gas atom.
  \[
  |\Psi\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}} e^{ikz} \Rightarrow \frac{1}{\sqrt{2}} \left[ e^{ikz} + \frac{\hbar}{r} f_1(\theta) \right]|1\rangle + \frac{1}{\sqrt{2}} \left[ e^{ikz} + \frac{\hbar}{r} f_2(\theta) \right]|2\rangle
  \]
- 2nd \(\pi/2\) pulse yields a Ramsey fringe.
- Probability current of state 2 has 3 terms: unscattered, interference, scattered.
  \[
  j = \frac{\hbar}{\mu} \text{Im} (\langle 2 | \nabla \langle 2 | \Psi \rangle) = \frac{\hbar k}{\mu} \cos^2 \left(\frac{\phi}{2}\right) + j_{2,\text{int}}(\theta = 0) + j_{2,\text{sc}}(\theta)
  \]
  \[
  j_{2,\text{int}} = \frac{\pi n\hbar}{\mu} \left\{ \text{Re} \left[ f_1(0) - f_2(0) \right] \sin(\phi) - 2 \text{Im} \left[ f_1(0) + f_2(0) \right] \cos^2 \left(\frac{\phi}{2}\right) \right\}
  \]

\(\theta=0\) dominates for ultracold atoms. with
\[
 f_1(\theta) = \sum_{\ell=0,\infty} \left(2\ell + 1\right) \sin(\delta_1) P_{2\ell}(\cos(\theta)) e^{i\delta_1} \sin(\phi) - 4 \left[ \sin^2(\delta_1) + \sin^2(\delta_2) \right] \cos^2 \left(\frac{\phi}{2}\right)
\]

\[
 j_{2,\text{int}} = \frac{\pi n\hbar}{2\mu k} \left\{ \sum_{\ell} \left(2\ell + 1\right) \left[ \sin(2\delta_{1\ell}) - \sin(2\delta_{2\ell}) \right] \sin(\phi) - 4 \left[ \sin^2(\delta_{1\ell}) + \sin^2(\delta_{2\ell}) \right] \cos^2 \left(\frac{\phi}{2}\right) \right\}
\]

KG, PRL '13
Ultracold vs. Room-temperature Clocks

\[ j_{2,\text{int}} = \frac{n\hbar}{2\mu k} \sum_{l} (2l + 1) \left[ \sin(2\delta_{1,l}) - \sin(2\delta_{2,l}) \right] \sin(\phi) \\
-4 \left[ \sin^2(\delta_{1,l}) + \sin^2(\delta_{2,l}) \right] \cos^2 \left( \frac{\phi}{2} \right) \]

- Large cross section for weak, long-range collisions
- Both clock states have nearly the same C_6 (in 2 slides).

\[ \delta_{2,l} = \delta_{1,l} + \Delta \delta \]
\[ \sin(2\delta_{1,l}) - \sin(2\delta_{2,l}) \approx -2\Delta \delta \cos(2\delta_{2,l}) \]

- Only weak, long-range background gas collisions shift the frequency of cold clock atoms. Averages to 0 for \( \delta_{2l} \approx 1 \).

For room-temperature clocks, \( j_{\text{sc}} + j_{\text{int}} \) give:

\[ j_{2,\text{hot}} = \frac{n\hbar}{2\mu k} \sum_{l} (2l + 1) \left[ \sin(2\delta_{1,l} - 2\delta_{2,l}) \sin(\phi) - 2\sin^2(\delta_{1,l} - \delta_{2,l}) \cos(\phi) \right] \]

Non-zero for \( \delta_{2l} \approx 1 \). Short and long-range collisions give a shift.

KG, PRL '13

Long-Range Van der Waals Collisions

\[ j_{2,\text{int}} = \frac{n\hbar}{2\mu k} \sum_{l} (2l + 1) \left[ \sin(2\delta_{1,l}) - \sin(2\delta_{2,l}) \right] \sin(\phi) \\
-4 \left[ \sin^2(\delta_{1,l}) + \sin^2(\delta_{2,l}) \right] \cos^2 \left( \frac{\phi}{2} \right) \]

- Small phase shifts are asymptotically: \( \delta_{l} = \frac{3\pi\mu C_{6} k^{4}}{16\hbar^{2} l^{5}} \)
- Analytic sum and thermal average:

\[ \langle j_{2,\text{int}} \rangle \approx -0.62n \left( \frac{2\mu k_{B} T}{m_{p}^{2}} \right)^{\frac{\gamma_{0}}{2}} \left[ \frac{C_{6}^{\frac{3}{2}}}{\hbar^{\frac{3}{2}}} \left[ \frac{\Delta C_{6}}{C_{6}} \sin(\phi) + 13.8\cos^2 \left( \frac{\phi}{2} \right) \right] \right] \]

- Very general, provided van der Waals phase shift is large.
  - Close for Cs-He and Cs-H_2.

1. Analytic frequency shift cross section: \( \langle j_{\text{shift}} \rangle \approx -0.62n \left( \frac{2\mu k_{B} T}{\hbar^{\frac{3}{2}} m_{p}^{2}} \right)^{\frac{\gamma_{0}}{2}} \left[ \frac{\Delta C_{6}}{C_{6}} \sin(\phi) \right] \)

Loss of fringe amplitude \( \Delta A \) scales in the same way.

- Evaluate uncertainty by measuring \( \Delta A \).
- Essentially independent of background gas.

\[ \frac{\Delta \nu}{\nu} \approx 2 \times 10^{-16} \Delta A \]

KG, PRL '13
In microwave clocks, both clock states have nearly identical $C_6$’s.
- 10% accuracy of $\Delta C_6$ is useful

$$C_6 = \frac{3}{2} \sum_{i,j} \frac{f_i f_{p,j}}{\Delta E_i \Delta E_{p,j} (\Delta E_i + \Delta E_{p,j})}$$

for Cs colliding with a perturber $p$.

- Oscillator strengths $f_i$ sum to 1 for both clock states.
- Smallest energies are the resonance energies $\Delta E_r$ and $\Delta E_{p,r}$.
  - Hyperfine splitting $h \nu$ is $10^{-4}$ of $\Delta E_1$.
- Cs has a lower resonance energy than common background gases (more polarizable).

$\Delta C_6/C_6$ ranges from $1/25,000$ (Cs) to $1/34,000$ (H$_2$).

Summary
- Unambiguously observe s-wave scattering phase shifts.
- Measured phase shifts are independent of the atomic density to 0th order.
  - Atomic-clock-like accuracy.
- Observe scattering phase shift through Feshbach resonances.
- Time variation of a Feshbach resonance.
  - Statistical uncertainty already allows 100 $\mu$rad precision.
  - Clock accuracy, sub-nG magnetic field
  - Need better understanding of Cs interactions.
- Background gas collision shift of cold atoms is due to van der Waal interactions.
  - $\Delta C_6/C_6$ is not suppressed in lattice clocks – 1nTorr gives $\approx 2.4 \times 10^{-18}$ ($\tau=6s$)

KG, PRL ’13