Atom Interferometry II

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LNE
Le progrès, une passion à partager

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IFRAF
Organization of the lecture

1: Applications of inertial sensors/gravimetry

2: A case study: the SYRTE atom gravimeter

3: Towards more compact sensors

4: Other instruments: gravimeters, gradiometers

5: Atomic gyroscope
Organization of the lecture

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Inertial sensors

**Inertial navigation** satellite, submarine...

**Fundamental physics**

- measurement of $\alpha, G$...
- watt balance (gravimeter)
- test of general relativity

Lense-Thirring effect (gyroscope)

WEP, anomalous gravity...(accelerometer)

gravitational waves

**Geophysics** ground or space

Earth’s rotation rate, tidal effects, gravity field mapping and monitoring
Gravimetry

✓ Deduce from local variations of gravity density variations in the ground

✓ Measure small variations with transportable sensors
  microgravimetry: $1 \mu \text{Gal} \sim 10^{-9} g$
  with $g \sim 9.8 \text{ m/s}^2 = 980 \text{ Gal}$

✓ Looking for small (temporal and local) changes $1 - 100 \mu \text{Gal}$

✓ Variation of $g$ with height: $3 \mu \text{Gal/cm}$
Simple examples in geophysics

✓ Measurement of (Bouguer) gravity anomaly to map influence of mass distribution, detect changes...

✓ Deformation and constraints:
  ➡ accumulation during seismic cycle...

✓ Volcanology: 4D mapping of mass distribution evolutions
Measurements at all scales
World gravity map

Earth's Gravity Field Anomalies (milligals)
### Table 1: Field of application, accuracy target, type of instruments and their availability.

<table>
<thead>
<tr>
<th>Application</th>
<th>Accuracy target</th>
<th>Instruments/availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isostasy</td>
<td>$10^{-5}$</td>
<td>Relative gravimeters (pendulum)/1817</td>
</tr>
<tr>
<td>Prospection for oil and gas</td>
<td>$10^{-6}$ — $10^{-7}$</td>
<td>Relative gravimeters/1950</td>
</tr>
<tr>
<td>Structural geology</td>
<td>$10^{-6}$</td>
<td>Relative gravimeters/1950</td>
</tr>
<tr>
<td>Microgravity prospection</td>
<td>$10^{-8}$</td>
<td>Relative gravimeters/1980</td>
</tr>
<tr>
<td>Metrology (forces)</td>
<td>$10^{-6}$ — $10^{-7}$</td>
<td>Absolute gravimeters/1980</td>
</tr>
<tr>
<td>Metrology (kilogram)</td>
<td>$10^{-8}$ — $10^{-9}$</td>
<td>Absolute gravimeters/1990</td>
</tr>
<tr>
<td>Geodetic metrology</td>
<td>$10^{-8}$</td>
<td>Absolute gravimeters/1980</td>
</tr>
<tr>
<td>Geodesy</td>
<td>$10^{-7}$ — $10^{-8}$</td>
<td>Rel + Abs gravimeters/1980</td>
</tr>
<tr>
<td>Earth tide and earth dynamic</td>
<td>$10^{-10}$ — $10^{-11}$</td>
<td>Superconducting gravimeters/1970</td>
</tr>
</tbody>
</table>

### Table 2: Progress in measuring $g$ (partially compiled after Torge 1989, page 14, figure 1.1 [1])

<table>
<thead>
<tr>
<th>Type of instrument</th>
<th>Time span</th>
<th>Uncertainty range</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reversible pendulum</td>
<td>1817 to 1945</td>
<td>$4 \times 10^{-5}$ to $2 \times 10^{-6}$</td>
<td>One order of magnitude/130 years</td>
</tr>
<tr>
<td>Spring type gravimeters</td>
<td>1935 to 2010</td>
<td>$4 \times 10^{-7}$ to $3 \times 10^{-9}$</td>
<td>Two orders of magnitude/75 years</td>
</tr>
<tr>
<td>Superconducting gravimeters</td>
<td>1965 to 2010</td>
<td>$5 \times 10^{-10}$ to $5 \times 10^{-12}$</td>
<td>Two orders of magnitude/45 years</td>
</tr>
<tr>
<td>Absolute gravimeters</td>
<td>1970 to 2010</td>
<td>$8 \times 10^{-7}$ to $2 \times 10^{-9}$</td>
<td>Two orders of magnitude/40 years</td>
</tr>
</tbody>
</table>
**Goal:** Measure Planck constant $h$ with a $10^{-8}$ relative accuracy

**Interest:** Replace the Kg etalon by a definition linked to $h$

### Static phase

- $B \cdot i \cdot L = m \cdot g$
- $F_{\text{Laplace}} = i \cdot L \cdot B$
- $B \cdot i \cdot L = m \cdot g$

### Dynamic phase

- $U = B \cdot L \cdot v$
- $m \cdot g \cdot v = U \cdot i$
- $m = k \frac{f_J^2}{g \times v} \cdot h$

**Accurate measurement of $g$ needed**

Quantum Hall and Josephson effects
Organization of the lecture

1: Applications of inertial sensors/gravimetry

2: A case study: SYRTE atom gravimeter

3: Towards more compact sensors

4: Other instruments: gravimeters, gradiometers

5: Atomic gyroscope
Interferometer configuration

3-Raman pulses interferometer
= Mach Zehnder/ Chu-Bordé interferometer
Acceleration phase shift

\[ \Phi(t) = \vec{k}_{eff} \cdot \vec{r}(t) \]

\[ \Phi_1(t_1) = 0 \]

\[ \Phi_2(t_2) = \frac{1}{2} \vec{k}_{eff} \cdot \vec{a} T^2 \]

\[ \Phi_3(t_3) = \frac{1}{2} \vec{k}_{eff} \cdot \vec{a} (2T)^2 \]

\[ \Delta \Phi = \Phi_1(t_1) - 2\Phi_2(t_2) + \Phi_3(t_3) = \frac{1}{2} \vec{k}_{eff} \cdot \vec{a} T^2 \]
Gravimeter: Implementation of Raman lasers

- **Vertical** Raman lasers
  \[ \Delta \Phi = k_{\text{eff}} \cdot g \cdot T^2 \]

- **Retroreflect** two (copropagating) Raman lasers
  Reduces influence of path fluctuations (common mode)
  \[ \Rightarrow 4 \text{ laser beams} \]
  \[ \Rightarrow 2 \text{ pairs of counterpropragating Raman lasers with opposite } k_{\text{eff}} \text{ wavevectors} \]

- Position of planes of equal phase difference attached to *position of retroreflecting mirror*

Interferometer measurement
  = relative displacement atoms/mirror

\[ z(T) = \frac{1}{2} g T^2 \]
\[ z(2T) = 2 g T^2 \]
SYRTE Gravimeter vacuum chamber

- Titanium vacuum chamber (non magnetic)
- 14 + 2 + 4 viewports
- Indium seals
- Pumps:
  - 2 × getter pumps 50 l/s
  - 1 × ion pump 2 l/s
  - 4 × getter pills
- Two layers magnetic shield
- Retroreflecting mirror under vacuum
Commercial fiber splitters
Fibered angled MOT collimators
Symmetric detection
Laser requirements

5 different frequencies: Cooling+Repumper + Raman 1&2 + Detection
Optical bench

« Compact » laser system : 60 x 90 cm
3 ECDL, 2 TA

**Key feature**: Use the same lasers for Cooling/Repumping and Raman beams.

Frequency control thanks to offset locks (using FVC) and/or PLL.
Sequence

• 2D + 3DMOT and far detuned molasses: ~ 100 ms  
  Loading rate ~ $10^9$ atoms/s  
  $10^8$ atoms at T~2μK

• State preparation: ~ 15 ms  
  Use of a combination of MW/pusher/Raman pulses  
  ~$10^7$ atoms in F=1, mF=0  
  $\delta v \sim 1\nu_r$

• Interferometer: 2T~140 ms  
  Raman pulses of typically ~10-20-10 μs

• Detection: ~ 80 ms

• Total cycle time: ~ 360 ms
Collection efficiency: 1%
Saturation parameter: \(2 \times 0.5\)
Vertical detection

Raman Vertical Beam back to resonance during the detection

1) First pulse at low intensity to slow down the F=2 atoms (cooling beam only)
2) Wait for the F=1 atoms to reach the bottom position
3) Second pulse at full power (cooling + repumper beams)
4) Third pulse for background subtraction

Collection efficiency : 1%
Saturation Parameter : 2 × 25
Fringe pattern

- Free fall $\rightarrow$ Doppler shift of the resonance condition of the Raman transition

$$\omega_1 - \omega_2 = \omega_e - \omega_f + \vec{k}_{\text{eff}} \cdot \vec{v}(t) + \frac{h k_{\text{eff}}^2}{2m} \quad \delta(\vec{v}) = \vec{k}_{\text{eff}} \cdot \vec{v} = \vec{k}_{\text{eff}} \cdot (\vec{g} t + \vec{v}_0)$$

- Ramping of the frequency difference to stay on resonance: $\omega = \omega_0 + \alpha t$

$$\omega = \omega_0 + \alpha t$$

$$\Delta \Phi = k_{\text{eff}} \cdot g \cdot T^2 - \alpha T^2$$
Principle of measurement, interferometer fringes

- Free fall $\rightarrow$ Doppler shift of the resonance condition of the Raman transition

$$\omega_1 - \omega_2 = \omega_e - \omega_f + k_{\text{eff}} \vec{v}(t) + \frac{\hbar k_{\text{eff}}^2}{2m}$$

- Ramping of the frequency difference to stay on resonance: $\omega = \omega_0 + \alpha t$

$$\Delta \Phi = k_{\text{eff}} g T^2 - \alpha T^2$$

![Graph showing interferometer fringes with probability of transition vs. Doppler shift and ramping frequency difference]
Principle of measurement: interferometer fringes

- Free fall \(\rightarrow\) Doppler shift of the resonance condition of the Raman transition

\[
\omega_1 - \omega_2 = \omega_e - \omega_f + \vec{k}_{\text{eff}} \vec{v}(t) + \frac{\hbar k_{\text{eff}}^2}{2m}
\]

- Ramping of the frequency difference to stay on resonance: \(\omega = \omega_0 + \alpha t\)

\[
\Delta \Phi = \vec{k}_{\text{eff}} \vec{g} T^2 - \alpha T^2
\]

- Dark fringe: independent of T

\[
g = \frac{\alpha_0}{k_{\text{eff}}}
\]

The measurement of \(g\) is a frequency measurement.
Principle of measurement: interferometer lock

\[ \alpha - \alpha_0 \]

\[ \alpha + \alpha_0 \]

\[ \delta \alpha = K (P^+ - P^-) \]

Digital Lock:
\[ \alpha \to \alpha + \delta \alpha \]
The sensitivity depends:

- on the noise on the measurement of $P$
- contrast of the fringes
- scaling factor ($k_{\text{eff}} T^2$)

\[
P = A + \frac{C}{2} \cos(\Delta \Phi)
\]

\[
\Rightarrow \delta \Phi = 2 \frac{\delta P}{C}
\]

\[
\Delta \phi = k_{\text{eff}} g T^2
\]

\[
\delta g = 2 \frac{\delta P}{k_{\text{eff}} C T^2}
\]
Parameters

\[ 2T = 140 \text{ ms} \]
\[ \tau = 20 \mu\text{s} \]
\[ \sigma_v \sim v_r \]
\[ N_{\text{det}} = 5 \times 10^6 \]
\[ T_c = 360 \text{ ms} \]

Contrast \( \sim 40 \text{-} 50 \% \)

Sources of noise
- Detection noise
- Laser phase noise
- Mirror vibrations

\[ \sigma_\Phi = \frac{1}{\text{SNR}} = 160 \text{ mrad/shot} \]
\[ \sigma_g/g \sim 2 \times 10^{-7} /\text{shot} \]
Detection noise

\[ \sigma_P^2 = \frac{A}{N^2} + \frac{1}{4N} + C \]

Electronic Noise
Quantum Projection Noise

\[ \sigma_\Phi = 2\sigma_P/C \]

C \sim 50\%

T = 70 ms & Tc= 360 ms

10^4 atoms : 40 mrad
5 \times 10^{-8} g/shot
3 \times 10^{-8} g at 1s

10^6 atoms : 2 mrad
2.5 \times 10^{-9} g/shot
1.5 \times 10^{-9} g at 1s
Sensitivity function $g(t)$

Quantifies the sensitivity of the interferometer to phase fluctuations

$$g(t) = 2 \lim_{\delta \phi \to 0} \frac{\delta P}{\delta \phi}$$

Hypothesis:
- $\delta \ll \Omega_R, \Omega_R \tau = \pi / 2$
- Plane waves
- Neglect spatial phase inhomogeneities at the scale of the separation between wavepackets
Sensitivity function $g(t)$

$$g(t) = 2 \lim_{\delta \phi \to 0} \frac{\delta P}{\delta \phi} \Rightarrow g_s(t) = \lim_{\delta \phi \to 0} \frac{\delta \Phi(\delta \phi, t)}{\delta \phi}$$

Method of calculation: Evolution matrix

$$\begin{pmatrix} C'_f(t_0 + \tau) \\ C_e(t_0 + \tau) \end{pmatrix} = M(t_0, \phi, \Omega_{eff}, \tau) \begin{pmatrix} C'_f(t_0) \\ C_e(t_0) \end{pmatrix}$$

Expression for the matrix:

<table>
<thead>
<tr>
<th>Raman pulse</th>
<th>Free evolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(t_0, \phi, \Omega_{eff}, \tau) =$</td>
<td>$M(T) = \begin{pmatrix} e^{-i\omega_f T} &amp; 0 \ 0 &amp; e^{-i\omega_e T} \end{pmatrix}$</td>
</tr>
</tbody>
</table>
| \[
\begin{pmatrix}
\cos \left( \frac{\Omega_{eff} |\tau|}{2} \right) e^{-i\omega_f \tau} & -ie^{i(\omega_1-\omega_2)t_0+\phi)} \sin \left( \frac{\Omega_{eff} |\tau|}{2} \right) e^{-i\omega_f \tau} \\
-ie^{-i((\omega_1-\omega_2)t_0+\phi)} \sin \left( \frac{\Omega_{eff} |\tau|}{2} \right) e^{-i\omega_e \tau} & \cos \left( \frac{\Omega_{eff} |\tau|}{2} \right) e^{-i\omega_e \tau}
\end{pmatrix}
\]| |

Total evolution in the interferometer: product of such matrices:

$M_{\text{interf}} = M(T + \tau, \phi_3, \Omega_{eff}, \tau)M(T)M(-\tau, \phi_2, \Omega_{eff}, 2\tau)M(T)M(-T - 2\tau, \phi_1, \Omega_{eff}, \tau)$

Result:

$$P = |C_e(T + 2\tau)|^2 = \frac{1 - \cos(\phi_1 - 2\phi_2 + \phi_3)}{2}$$
Sensitivity function $g(t)$

In the case of a phase jump:

$$\varphi$$

\[ \uparrow \delta \varphi \]

$$t$$

Split a matrix in two:

Example if the phase jump occurs during the first pulse

$$M_{t,\delta \varphi}(-T-2\tau, \phi, \Omega_{eff}, \tau) = M(t, \phi+\delta \phi, \Omega_{eff}, -T-\tau-t)M(-T-2\tau, \phi, \Omega_{eff}, t+T+2\tau)$$

Result (for $t>0$):

$$g(t) = \begin{cases} 
\sin(\Omega_R t) & 0 < t < \tau \\
1 & \tau < t < T + \tau \\
\sin(\Omega_R (t-T)) & T + \tau < t < T + 2\tau 
\end{cases}$$
The sensitivity function is an odd function
Experimental demonstration

Response to a time dependent perturbation

Time dependent phase fluctuations

\[ \varphi(t) \Rightarrow P = \frac{1}{2} \int g(t) \frac{d\varphi}{dt} \, dt \]

Example: gravity !!:

\[ \frac{d\varphi}{dt} = k_{\text{eff}} \frac{dz}{dt} = k_{\text{eff}} v = k_{\text{eff}} (gt + v_0) \]

Gives the scale factor:

\[ \Delta \phi = k_{\text{eff}} g \cdot (T + 2\tau) \cdot (T + \frac{4\tau}{\pi}) \]

In the spectral domain:

\[ \sigma_{\phi}^2 = \int G(\omega)^2 \omega^2 S_\varphi(\omega) \frac{d\omega}{2\pi} = \int H(\omega)^2 S_\varphi(\omega) \frac{d\omega}{2\pi} \]

\[ H(\omega) = -\frac{4i \omega \Omega_R}{\omega^2 - \Omega_R^2} \sin(\frac{\omega(T + 2\tau)}{2})(\cos(\frac{\omega(T + 2\tau)}{2}) + \frac{\Omega_R}{\omega} \sin(\frac{\omega T}{2})) \]
Transfer function $H(\omega)$

\[
H(\omega) = -\frac{4i\omega\Omega_R}{\omega^2 - \Omega_R^2} \sin\left(\frac{\omega(T + 2\tau)}{2}\right) \cos\left(\frac{\omega(T + 2\tau)}{2}\right) + \frac{\Omega_R}{\omega} \sin\left(\frac{\omega T}{2}\right)
\]

Insensitive to frequencies multiples of $1/(T+2\tau)$

Finite duration of the pulses
Low pass filtering
Stability characterized by the Allan standard deviation

Sampled measurement: $f_c = 1/T_c \Rightarrow$ Aliasing

For long enough averaging time, only components at frequencies at multiples of the cycling frequency contribute

\[
\sigma^2_{\Phi}(\tau_m) = \frac{1}{\tau_m} \sum_{n=1}^{\infty} \left| H(2\pi nf_c) \right|^2 S_{\phi}(2\pi nf_c)
\]

White phase noise ($S^0_{\phi}$) $\Rightarrow$ \[
\sigma^2_{\Phi}(\tau_m) = \left( \frac{\pi}{2} \right)^2 \frac{S^0_{\phi}}{\tau} \frac{T_c}{\tau_m}
\]
Derivation of the stability

Allan standard deviation

\[ \sigma^2_{\Phi}(\tau_m) = \frac{1}{2} \lim_{n \to \infty} \left\{ \frac{1}{n} \sum_{k=1}^{n} (\delta \Phi_{k+1} - \delta \Phi_k)^2 \right\} \]

where \( \delta \Phi_k \) is the \( k \)-th average of the interferometer phase over duration \( \tau_m \)

Using the sensitivity function, it can be expressed as

\[
\bar{\delta \Phi}_k = \frac{1}{m} \sum_{i=1}^{m} \int_{t_k + (i-1)T_c}^{t_k + iT_c} g_s(t - t_k - (i - 1)T_c - T_c/2) \frac{d\phi}{dt} \, dt \\
= \frac{1}{m} \int_{t_k}^{t_k+1} g_k(t) \frac{d\phi}{dt} \, dt
\]

where

\[ g_k(t) = \sum_{i=1}^{m} g_s(t - kmT_c - (i - 1)T_c) \]
Derivation of the stability

The difference between consecutive averages can be expressed as:

$$\overline{\delta \Phi}_{k+1} - \overline{\delta \Phi}_k = \frac{1}{m} \int_{-\infty}^{+\infty} (g_{k+1}(t) - g_k(t)) \frac{d\phi}{dt} \, dt$$

Assuming uncorrelated averaged samples

$$\sigma^2_{\Phi}(\tau_m) = \frac{1}{2} \frac{1}{m^2} \int_{0}^{+\infty} \left| \omega G_m(\omega) \right|^2 S_{\phi}(\omega) \frac{d\omega}{2\pi}$$

with \( |G_m(\omega)|^2 = \frac{4 \sin^4(\omega m T_c/2)}{\sin^2(\omega T_c/2)} |G(\omega)|^2 \)

As \( \lim_{\tau \to \infty} |G_m(\omega)|^2 = \frac{2m}{T_c} \sum_{j=-\infty}^{\infty} \delta(\omega - j2\pi f_c) |G(\omega)|^2 \)

We finally get

$$\sigma^2_{\Phi}(\tau_m) = \frac{1}{\tau_m} \sum_{n=1}^{\infty} |H(2\pi nf_c)|^2 S_{\phi}(2\pi nf_c)$$
The sensitivity function is a simple and powerful tool

Allows to calculate the influence of all kinds of perturbations:

- Laser/microwave reference phase/frequency noise
- Magnetic field fluctuations
- Light shift fluctuations
- Vibrations
- ...
The two Raman lasers are phase locked onto an ultra-low noise \(\mu\)wave oscillator.

The \(\mu\)wave oscillator is phase locked onto an ULN Quartz oscillator.

The Raman phase is also perturbed by non common perturbations during propagation.
Laser phase stability: influence of ULN Quartz

100 MHz reference signal realized by phase locking an ULN 100 MHz oscillator onto an ULN 5 MHz oscillator

\[
\sigma_{\Phi}^2(\tau_m) = \frac{1}{\tau_m} \sum_{n=1}^{\infty} |H(2\pi n f_c)|^2 S_\varphi(2\pi n f_c)
\]

Contribution to the noise of the interferometer dominated by low frequencies

T = 70 ms

Total: \(6.4 \times 10^{-7}\)

\(\Rightarrow 0.8 \times 10^{-3}\) rad @1s

\(\Rightarrow \sim 1\) µGal @1s
Influence of laser phase noise

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma_{\Phi}$ (mrad/coup)</th>
<th>$\sigma_g$ (g/Hz$^{1/2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Référence 100 MHz</td>
<td>1,0</td>
<td>1,3·10^{-9}</td>
</tr>
<tr>
<td>Synthèse HF</td>
<td>0,7</td>
<td>0,9·10^{-9}</td>
</tr>
<tr>
<td>Résidu PLL</td>
<td>1,6</td>
<td>2,0·10^{-9}</td>
</tr>
<tr>
<td>Fibre optique</td>
<td>1,0</td>
<td>1,3·10^{-9}</td>
</tr>
<tr>
<td>Rétro-réflexion</td>
<td>2,0</td>
<td>2,6·10^{-9}</td>
</tr>
<tr>
<td>Total</td>
<td>3,1</td>
<td>3,9·10^{-9}</td>
</tr>
<tr>
<td>Détection</td>
<td>4</td>
<td>5·10^{-9}</td>
</tr>
<tr>
<td>Total {lasers+détection}</td>
<td>5,0</td>
<td>6,5·10^{-9}</td>
</tr>
</tbody>
</table>

(Evaluation of noise sources for T=50 ms)

Vibration noise

A fluctuation $\delta z$ of the lasers equiphase induces a change $\delta \phi = k_{\text{eff}} \delta z$

As $S_\phi(\omega) = k_{\text{eff}}^2 S_z(\omega) = k_{\text{eff}}^2 \frac{S_a(\omega)}{\omega^4}$

the contribution to the stability can be expressed as

$$
\sigma_\Phi^2(\tau_m) = \frac{k_{\text{eff}}^2}{\tau_m} \sum_{n=1}^{\infty} \frac{|H(2\pi n f_c)|^2}{(2\pi n f_c)^4} S_a(2\pi n f_c)
$$

2T=100 ms

Transfer function for acceleration noise

⇒ Sensitivity to low frequencies
Vibration isolation

\[
\sigma_g^2(\tau) = \frac{1}{\tau} \sum_{k=1}^{\infty} \left( \frac{\sin(\pi k f_c T)}{\pi k f_c T} \right)^4 S_a(2\pi k f_c) \]

Passive isolation platform:
Natural frequency 0.5 Hz

Acoustic + air flow isolation

Corresponding sensitivities @ 1s:
2.9 \cdot 10^{-6} \text{ g} \quad \text{when OFF (day)}
7.6 \cdot 10^{-8} \text{ g} \quad \text{when ON (day)}

=> Gain : factor 40
Residual vibration corrections

- Measure vibrations with a seismometer at the same time
- Convert the seismometer signal into an interferometer phase shift

\[ \phi_{vib}^S = k_{eff} K_s \int_{T}^{T-T} g_s(t) U_s(t) \, dt \]

Good correlations with the calculated phase shift

⇒ the seismometer is used to correct the atomic measurement
Residual vibration corrections

Typical sensitivity
\[ \sigma_g/g = 2 \times 10^{-8} \text{ at } 1\text{s} \]

Resolution \( \sim 10^{-9}g \) after 500-1000 s

Gain: from 2 to \( \sim 10 \)

Seismometer \( \rightarrow \phi_{vib} \rightarrow \phi_{vib}^S \)

PC \( \rightarrow k_{eff}gT^2 \)

Interferometer

\[ k_{eff}gT^2 + \phi_{vib}^S \]
Seismometer transfer function

High pass filter: 30 s (0.03 Hz)
Low pass filter (mechanical resonances): $f_c = 50$ Hz

Phase shift between seismometer signal and mirror vibrations

→ Rejection at high frequency will be limited

**Correction:**

$$\Phi_c(\omega) = \Phi(\omega) - H(\omega)\Phi(\omega) = (1 - H(\omega))\Phi(\omega)$$

**Gain:**

$$G(\omega) = \frac{\Phi(\omega)}{\Phi_c(\omega)} = \frac{1}{1 - H(\omega)}$$
Non linear filtering

Implement a digital filter

Goal: compensate the phase shift of the seismometer

\[ F(f) = \frac{1 + jf/f_0}{1 + jf/f_1} \frac{1}{1 + (f/f_c)^2} \]

Expected gain of a factor of \( \sim 3 \) \( \Rightarrow \sigma_g < 10^{-8} \, g \, \text{Hz}^{-1/2} \)
Filtre NL, results

Best short term sensitivity: $1.4 \times 10^{-8} \text{ g @ 1s}$

Modest gain (<expected 3 )

Self noise of the seismometer: $4 \times 10^{-9} \text{ g.Hz}^{-1/2}$ (according specs)

Cross-couplings:
→ Acquisition of 3 axes is necessary

Alternative technique (simpler) : compensate for the delay

Same results when implementing a delay of 4.6 ms
Acquisition without isolation platform

\[ v(t) \rightarrow \varphi^{S}_{\text{vib}} \]

Post corrections

\[ k_{\text{eff}} g T^2 + \varphi^{S}_{\text{vib}} \]

Influence of the filtering much larger in the presence of large vibration noise

Makes measurements feasible in noisy environment:
- outside the lab
- in mobile vehicle

Fringe fitting

Post correction of mirror vibrations with

$$\phi_{vib}^s = k_{eff} K_s \int_T^{-T} g_s(t) U_s(t) \, dt$$

Fringe fitting and mod ±π/2

$$P = a + b \cos( \sum_{j=x,y,z} \eta_j \phi_{vib,j}^s + \delta\phi)$$

In the presence of high anthropic noise, we have reached gains as high as 100 !!
1- Some effects are $k_{\text{eff}}$ orientation dependent

\[
\Delta \Phi_{\text{int}} = -k_{\text{eff}} \cdot g T^2 + \Delta \Phi_{LS2} + \Delta \Phi_{\text{Coriolis}} + \Delta \Phi_{Ab} + \Delta \Phi_{RF} + \Delta \Phi_{LS1} + \Delta \Phi_{\text{Zeeman}}
\]

Dependent of $k_{\text{eff}}$ orientation

Independent

\[
|f,p\rangle \quad \downarrow \quad g \quad \downarrow
\]
1- Some effects are $k_{\text{eff}}$ orientation dependent

$$\Delta \Phi_{\text{int}} = -k_{\text{eff}} \cdot g T^2 + \Delta \Phi_{LS2} + \Delta \Phi_{\text{Coriolis}} + \Delta \Phi_{\text{Ab}} + \Delta \Phi_{RF} + \Delta \Phi_{LS1} + \Delta \Phi_{\text{Zeeman}}$$

Dependent of $k_{\text{eff}}$ orientation

$$\frac{\Delta \Phi}{2} = -k_{\text{eff}} \cdot g T^2 + \Delta \Phi_{\text{dependent}}$$

→ Reject independent effects by switching between $k_{\text{eff}} \uparrow/\downarrow$: need two measurements
1- Some effects are $k_{\text{eff}}$ orientation dependent

$$\Delta \Phi_{\text{int} \uparrow} = -k_{\text{eff}} \cdot g T^2 + \Delta \Phi_{LS2} + \Delta \Phi_{\text{Coriolis}} + \Delta \Phi_{Ab} + \Delta \Phi_{RF} + \Delta \Phi_{LS1} + \Delta \Phi_{\text{Zeeman}}$$

Dependent of $k_{\text{eff}}$ orientation

Independent

2- $\Delta \Phi_{LS2} \propto \Omega_{\text{eff}}$ → Reject $\Delta \Phi_{LS2}$ bias alternating measurements $\Omega_{\text{eff}}$ and $\Omega_{\text{eff}}/2$

$$\Delta \Phi_{\uparrow,\Omega/2} - \Delta \Phi_{\downarrow,\Omega/2} - \frac{\Delta \Phi_{\uparrow,\Omega} - \Delta \Phi_{\downarrow,\Omega}}{2} = k_{\text{eff}} g T^2 + \Delta \Phi_{\text{Coriolis}} + \Delta \Phi_{Ab}$$

The algorithm degrades the sensitivity by $\sqrt{10}$
12 days of continuous measurement

Gal ?

1 μGal
= 10^{-8} \text{ m.s}^{-2}
≈ 10^{-9} g

Gray: 400 drops (170 s)
Black: 10 000 s
Sensitivity flickers at \(7-8 \times 10^{-10} \, g\)

Limited by tidal model? Water table? Soil moisture? Instrument?
Coriolis acceleration

For $T = 2 \mu K$, $\sigma_v \sim 1 \text{ cm.s}^{-1} = 100 \upsilon_0$ !

Average shift is null if
1) velocity distribution symmetric
2) detection symmetric
3) mean velocity is zero

Solutions
Control $\upsilon_0$ (CCD imaging, Raman velocimetry/selection)
Colder atoms
Alternate $\pm \upsilon_0$ (rotate the experiment by 180°)

$\Delta \Phi = \vec{k}_{\text{eff}} T^2 \cdot (2\vec{\Omega} \wedge \vec{v})$

$\Delta g = 10^{-9} g$ for $\upsilon_0 = 100 \mu \text{m.s}^{-1}$

Non zero transverse velocity $\rightarrow$ sensitivity to the Sagnac effect
Coriolis acceleration

- Handle on the mean velocity: **power imbalance EO molasses beams**
- Alternate two opposite orientations **within 2 hours**

- Opposite trends for the two orientations, **signature of Coriolis shift**

- Trend is not so linear
  - **changes in the aberration shift?**
  - **why linear anyway?**

- At the crossing point: **Null (mean) Coriolis shift**

![Graph showing Coriolis acceleration with data points for ORIENTATION N and ORIENTATION S.](image)
**Coriolis acceleration**

Changes in Coriolis Vs changes in the wavefront distortions shift?

**Half difference : Coriolis**

**Half sum : changes in the aberration shift**

---

**Graph:**

- **Null Coriolis bias**

- **Imbalance in the molasses beams** $i = (P2-P1)/(P2+P1)$

- **Uncertainty in the correction of the Coriolis shift**: $0.4 \, \mu\text{Gal}$
Wavefront aberrations

Wavefronts are not flat: gaussian beams, flatness of the optics ...

Case of a curvature

\[ \delta \phi = K \cdot r^2 \text{ (with } K = k_1/2R) \]

\[ \Delta \Phi_{ab} = 2K \sigma_v^2 T^2 = \frac{2k_1 k_B T_{at}}{R m} T^2 \]

\[ \Delta g < 10^{-9} \text{ g with } T = 2 \, \mu K \]

\[ \rightarrow R > 10 \text{ km !} \]

\[ \rightarrow \text{ flatness better than } \lambda/300 !!! \]

Measure aberrations
with wavefront sensor
+ excellent optics
+ colder atoms
Characterization of the optics

- Mirror
  - 40mm diameter
  - $PV = \frac{\lambda}{10}$
  - $RMS = \frac{\lambda}{100}$

Simulation:
- $T = 2.5 \mu K$
- $\sigma = 1.5 mm$

- $\frac{\delta g}{g} = 1.4 \times 10^{-9}$
- $\frac{\delta g}{g} = 8 \times 10^{-9}$
Mirror + 1/4 plate under vacuum
Exploration of the wavefronts:
Differential measurements for different cloud temperatures and extrapolation to zero

Analysis with a set of Zernike polynomials => No reliable extrapolation

Need to limit the expansion of the cloud:
- colder atoms and/or use horizontal Raman selection
Self gravity effect

\[ \Delta g = (1.3 \pm 0.1) \times 10^{-8} \text{ m.s}^{-2} \]

G. D'Agostino et al, Metrologia 48 (2011)
## Accuracy budget

<table>
<thead>
<tr>
<th>Effect</th>
<th>Bias</th>
<th>Uncertainty</th>
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<td>$v_{gg}$</td>
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<td><strong>TOTAL</strong></td>
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