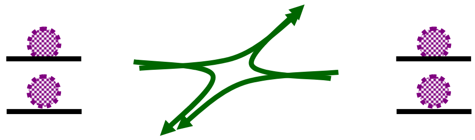
Distinguishable Fermions?

Identical?

$$\frac{|\uparrow\rangle+|\downarrow\rangle}{\sqrt{2}}$$



$$\frac{|\uparrow\rangle-|\downarrow\rangle}{\sqrt{2}}$$

Despite decoherence, Bosons and Fermion often appear to be quantum mechanically identical.

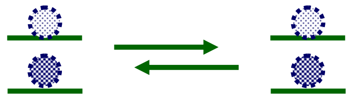
Ultracold Fermions

- At ultracold temperatures, only s-wave scattering.
- Antisymmetric wavefunction → no scattering of identical superpositions!

$$\sigma = 0$$

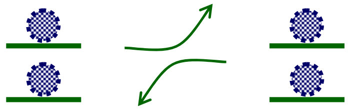
$|\uparrow\rangle$

$|\downarrow\rangle$

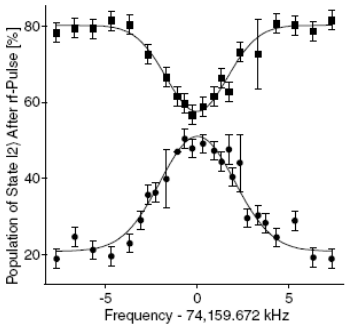


- Decoherence → distinguishable fermions → collisions

$$\sigma \neq 0 \quad \frac{|\uparrow\rangle+|\downarrow\rangle}{\sqrt{2}}$$

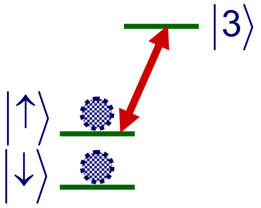


$$\frac{|\uparrow\rangle-|\downarrow\rangle}{\sqrt{2}}$$



- No Shift! Despite being distinguishable, fermions act as if they're indistinguishable!
- $[H_{light}, V_{\uparrow\downarrow}]=0$ →

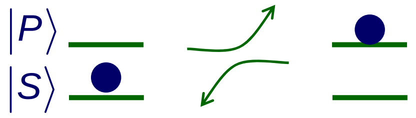
Fermions are Universally Immune to Collisions!



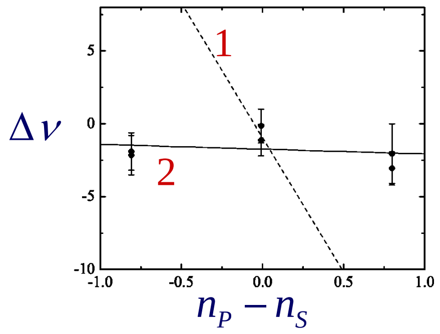
Zwierlein, Hadzibabic, Gupta, & Ketterle, PRL '03

Bosons & the Factor of 2 Controversy

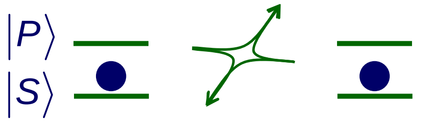
- Identical bosons in different states are distinguishable.



$$\sigma = 1 \times 4\pi a_{SP}^2$$

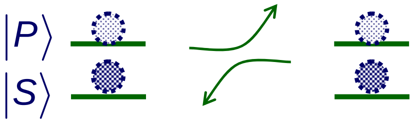


- Exchange symmetry of identical bosons.



$$\sigma = 2 \times 4\pi a_{SS}^2$$

- Exchange symmetry of identical superpositions.



$$\sigma = 2 \times 4\pi a_{SP}^2$$

“The Mystery of the Ramsey Fringe that Didn’t Chirp”

- Decoherence destroys interstate coherence.
  - No exchange symmetry

$$\frac{m}{2\hbar V} \times \Delta\nu = n\left(a_{PP} - a_{SS}\right) + \left(\boxed{1}\,a_{SP} - a_{PP} - a_{SS}\right)\left(n_P - n_S\right)$$

Harber, Lewandowski, McGuirk, & Cornell, PRA ’02, ICAP ‘02

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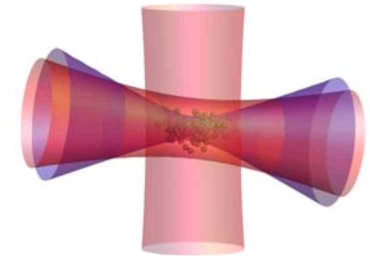
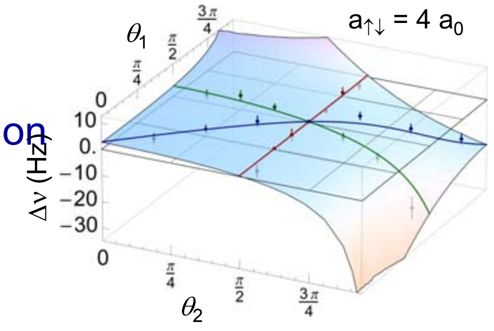
s-wave Frequency Shifts & Spin Waves in Fermion Clocks



- s-wave frequency shift of 2 fermions
- Experiments: ^{87}Rb Chip & ^{6}Li Fermion “Clock”
- Observation of distinguishing characteristics
- Tune scattering length near an s-wave Feshbach resonance
- Strong Interactions – coherence & collision shifts
- Scattering-induced tunneling.
- Spin waves



Penn State
Eric Hazlet
Kunyun Ye
Ron Stites
Kurt Gibble
Ken O’Hara

Christian Deutsch
Jakob Reichel

Obs. Paris
Wilfried Maineult
Peter Rosenbusch

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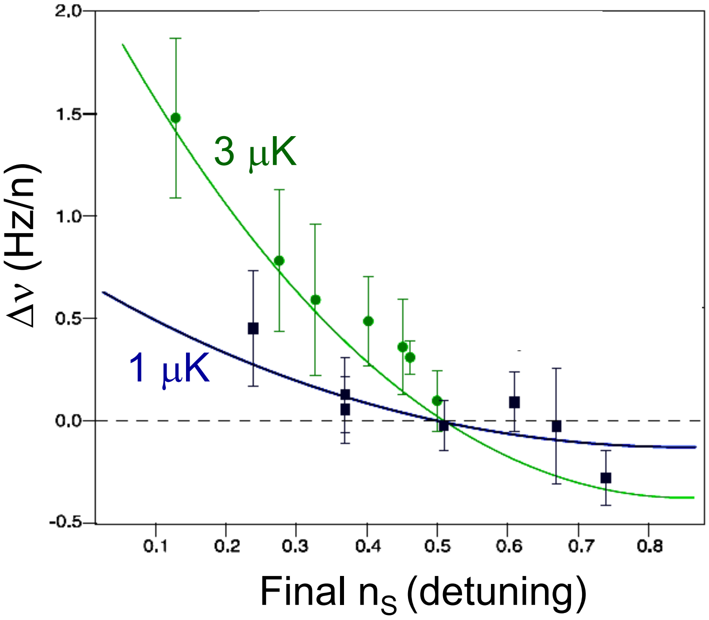
Optical Clock Fermion Collisions

- “Fermions are universally immune to collisions.”
- Campbell & Ye observed a collision shift for fermions.
- Treated shift as ∝ $n_S - n_P$.

$$\Delta\nu \overset{?}{=} \frac{2\hbar a_{SP}}{m} g^{(2)}\left(n_S - n_P\right)$$

- $[H_{light}, V_{SP}] \neq 0$ for clock fields with fast spatial variations.
- Observed Sr & Yb shifts are p-wave

:

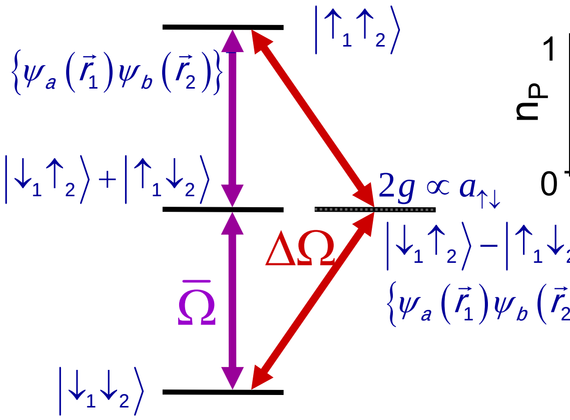


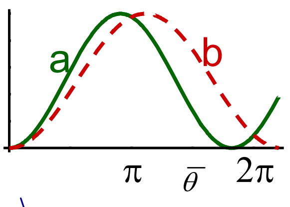
Zwierlein, Hadzibabic, Gupta, & Ketterle, PRL ‘03 Lemke, … Oates, PRL ‘09
Campbell, … Julienne, Ye, Science ‘09 Lemke, … Oates, Ludlow, PRL ‘11

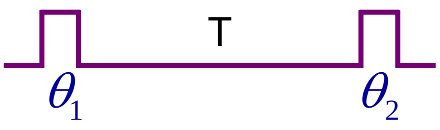
Fermion Clock Collision Shift

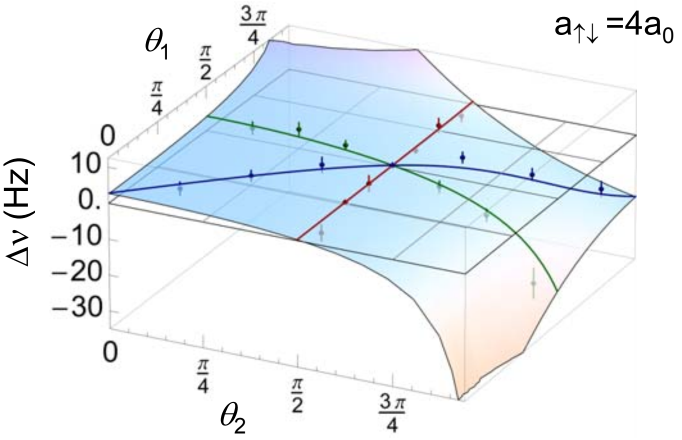
- Many particles are sum of pair-wise effects
- Basis – Singlet and Triplet states of 2 atoms:

$\Omega = \Omega_a e^{i\Delta\omega_a t}$ $\Omega' = \Omega_b e^{i\Delta\omega_b t}$









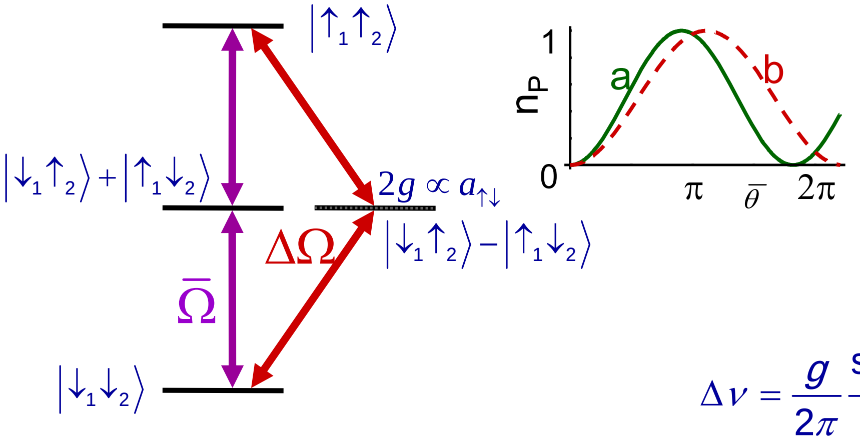
$$\Delta\nu = \frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

KG, PRL ‘09
Hazlett, Zhang, Stites, KG, O’Hara PRL ‘13

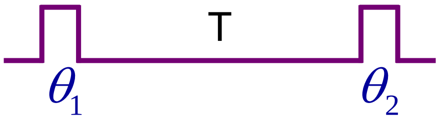
l'Observatoire de Paris SYRTE Systèmes de Référence Temps-Espace

PENNSTATE 1855

Fermion Clock Collision Shift



$$\Omega=\Omega_a e^{i\Delta\omega_a t} \qquad \Omega'=\Omega_b e^{i\Delta\omega_b t}$$



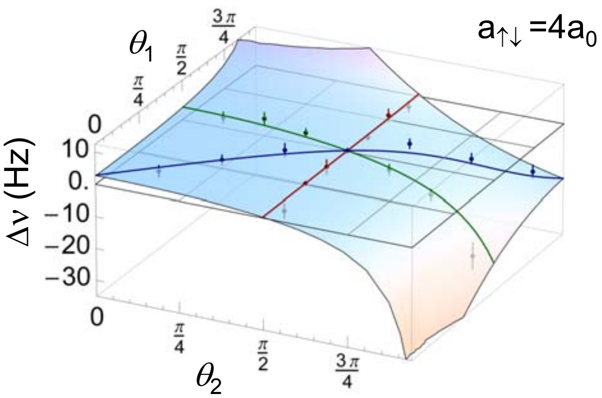
$$\Delta\nu=\frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

- $\Delta\theta_1=0 \Rightarrow$ s=0; identical fermions
  - $\Delta\nu$ is not proportional to n_S-n_P (excitation fraction)

$$\cos(\bar{\theta}_1)=\frac{n_S-n_P}{n}$$

- No shift for $\Delta\theta_2=0$
- No shift for $\bar{\theta}_2=\pi/2$

$a_{\uparrow\downarrow}=4a_0$



KG, PRL '09 Hazlett, Zhang, Stites, KG, O'Hara PRL '13

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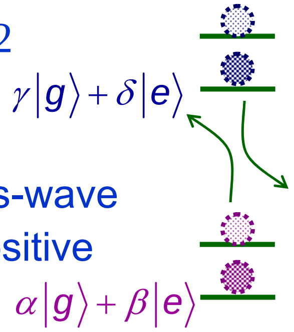
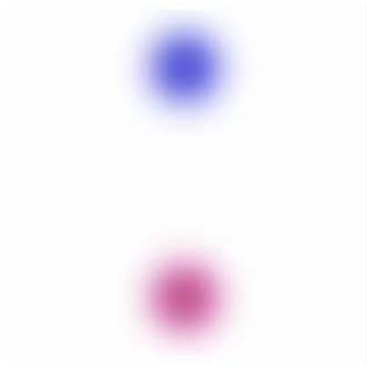
l'Observatoire de Paris SYRTE Systèmes de Référence Temps-Espace

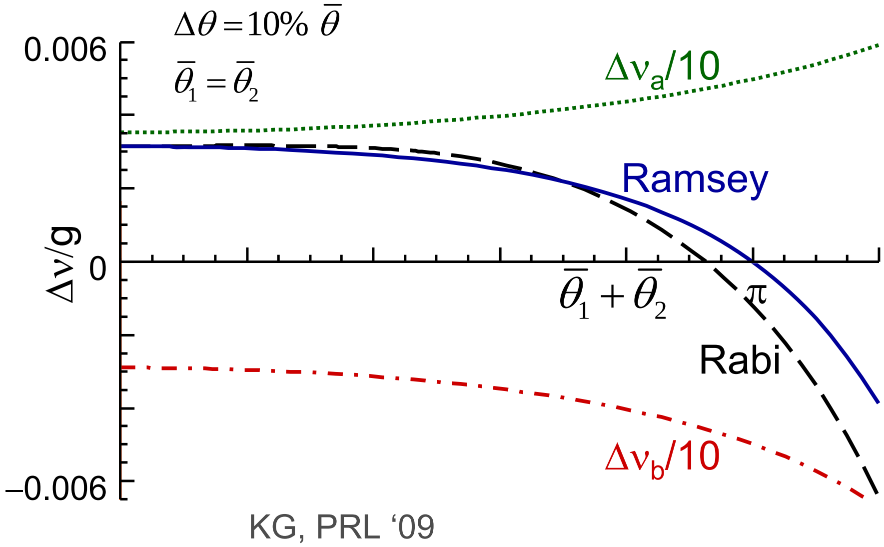
PENNSTATE 1855

Optical Clock Fermion Collisions

- No collision shift for $\bar{\theta}_2=\pi/2$
- Connect to free space scattering.
- Entangled S & P scattered s-wave
- One particle gets a large positive shift, the other negative







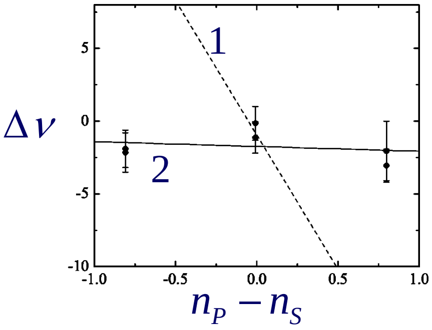
Rabi is the same as Ramsey.
Absence of shift is due to Bloch rotation readout of phase.

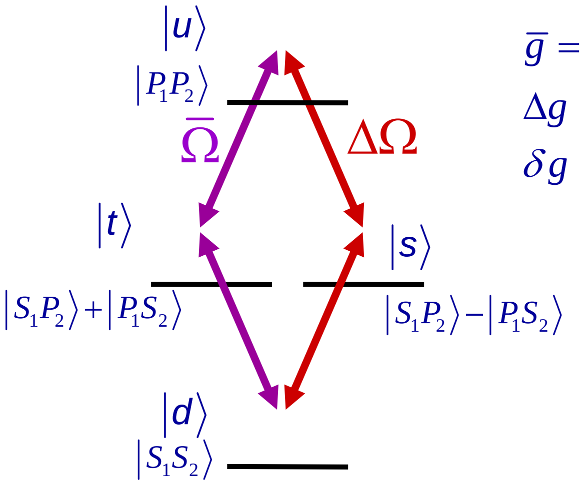
KG, PRL '09

Hart, Xu, Legere, & KG, Nature '07

“2” & “^{87}Rb is a fermion” in a clock

- All 3 scattering lengths are nearly equal.
  - Overall energy shift of triplet states
- 2 is always 2.
- Decoherence appears via singlet states





$$\bar{g} = \tfrac{1}{2}\left(g_{PP} + g_{SS}\right)$$
$$\Delta g = g_{PP} - g_{SS}$$
$$\delta g = 2g_{SP} - g_{PP} - g_{SS}$$

$$i\dot{d} = \frac{\bar{\Omega}}{\sqrt{2}}t - \frac{\Delta\Omega}{\sqrt{2}}s - \Delta g d$$
$$i\dot{t} = \frac{\bar{\Omega}}{\sqrt{2}}u + \frac{\bar{\Omega}^*}{\sqrt{2}}d + \delta g t$$
$$i\dot{s} = -\frac{\Delta\Omega^*}{\sqrt{2}}d + \frac{\Delta\Omega}{\sqrt{2}}u - 2\bar{g}s$$
$$i\dot{u} = \frac{\bar{\Omega}^*}{\sqrt{2}}t + \frac{\Delta\Omega^*}{\sqrt{2}}s + \Delta g u$$

$$2\pi\Delta\nu = n\left(g_{PP} - g_{SS}\right) + \left(\boxed{1}g_{SP} - g_{PP} - g_{SS}\right)\left(n_P - n_S\right) - n\bar{g}\frac{\sin\left(2\Delta\theta_1\right)\sin\left(\Delta\theta_2\right)\cos\left(\bar{\theta}_2\right)}{2\sin\left(\bar{\theta}_1\right)\sin\left(\bar{\theta}_2\right)}$$

Harber, Lewandowski, McGuirk, & Cornell, PRA ‘02

Boson & Fermion Collision Shifts

Homogeneous Excitations | Inhomogeneous

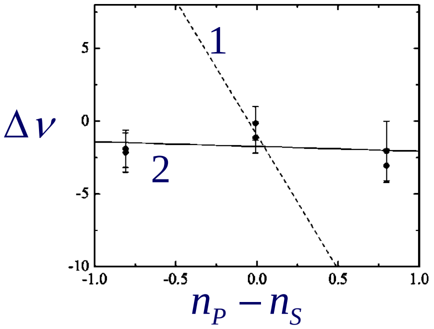
Fermions (p-wave bosons):

$$\Delta\nu = \frac{2\hbar}{m}\left[\quad 0 \quad + \left(a_{PP} + a_{SS}\right)\left(n_P + n_S\right)\frac{\sin\left(2\Delta\theta_1\right)\sin\left(\Delta\theta_2\right)\cos\left(\bar{\theta}_2\right)}{4\sin\left(\bar{\theta}_1\right)\sin\left(\bar{\theta}_2\right)}\right]$$

Bosons (p-wave fermions):

$$\Delta\nu = \frac{2\hbar}{m}\left[\left(a_{PP} - a_{SS}\right)\left(n_P + n_S\right) + \left(\boxed{2}a_{SP} - a_{PP} - a_{SS}\right)\left(n_P - n_S\right)\right]$$

$$-\left(a_{PP} + a_{SS}\right)\left(n_P + n_S\right)\frac{\sin\left(2\Delta\theta_1\right)\sin\left(\Delta\theta_2\right)\cos\left(\bar{\theta}_2\right)}{4\sin\left(\bar{\theta}_1\right)\sin\left(\bar{\theta}_2\right)}\Bigg]$$



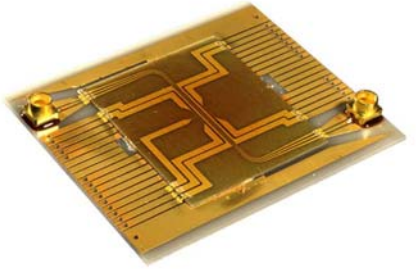
A number of papers report density shift versus excitation fraction (Rabi pulse).

Harber, Lewandowski, McGuirk, & Cornell, PRA ‘02

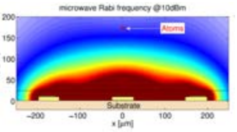
KG, PRL ‘09

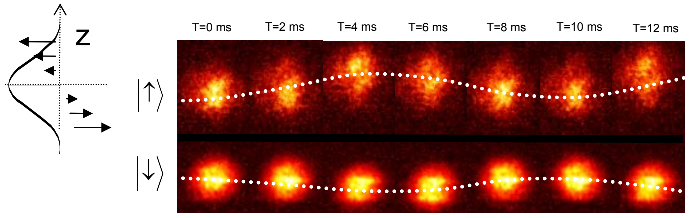
Optical Clock Fermion Collisions - Simulation with Microwave Boson Clock

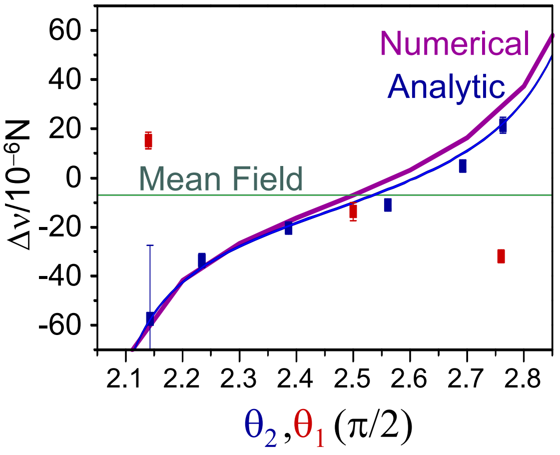
- ^{87}Rb spin-exchange is almost mathematically identical to fermions.
- Excite dipolar spin waves.



$$\Omega(z) = \Omega_0\left(1 + \delta_1 z + \ldots\right)$$







$$\Delta\nu = -\frac{\sin\left(gT_R\right)}{T_R}\frac{\cos\left(\omega_z T_R\right)}{4\pi}\frac{\Delta\theta_1\Delta\theta_2\cos\left(\bar{\theta}_2\right)}{\sin\left(\bar{\theta}_1\right)\sin\left(\bar{\theta}_2\right)}$$

- Sideband excitations

Maineult, Deutsch, KG, Reichel, Rosenbusch PRL '12

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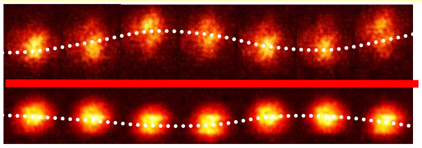
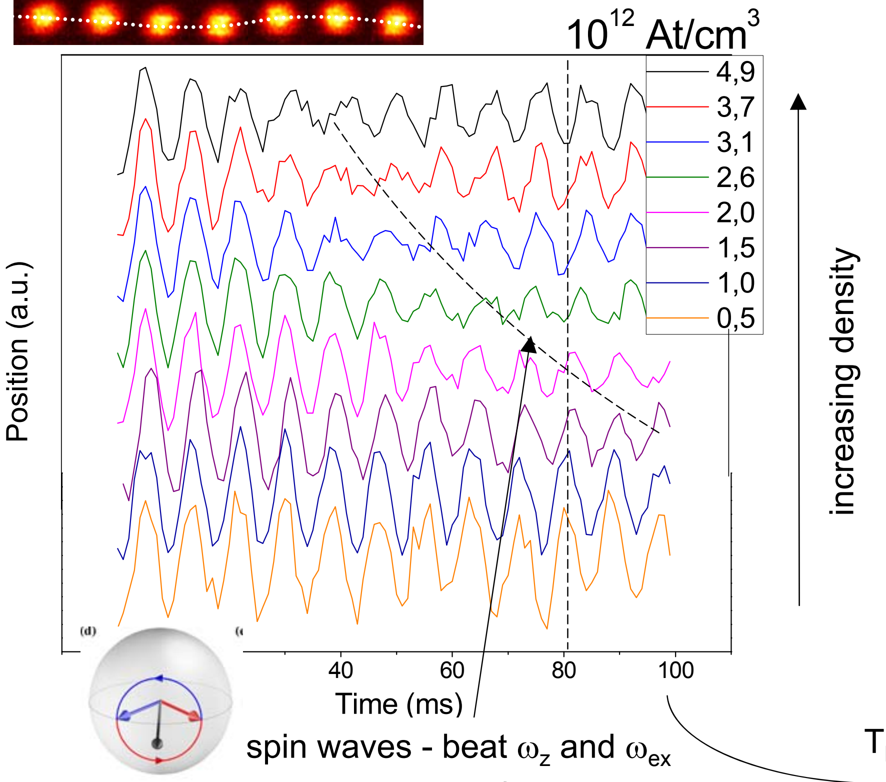
spin waves





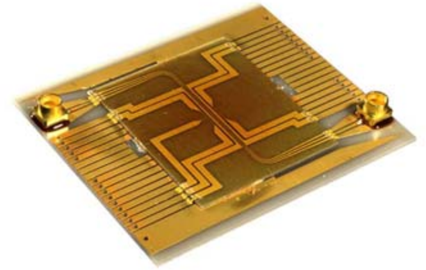
spin waves - beat ω_z and ω_{ex}

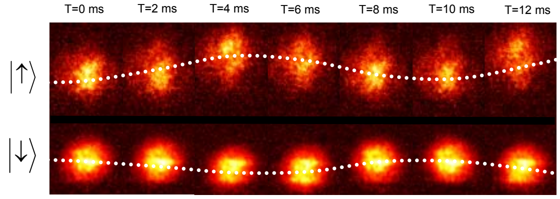
→ fit ω_{ex}

T_R=100ms

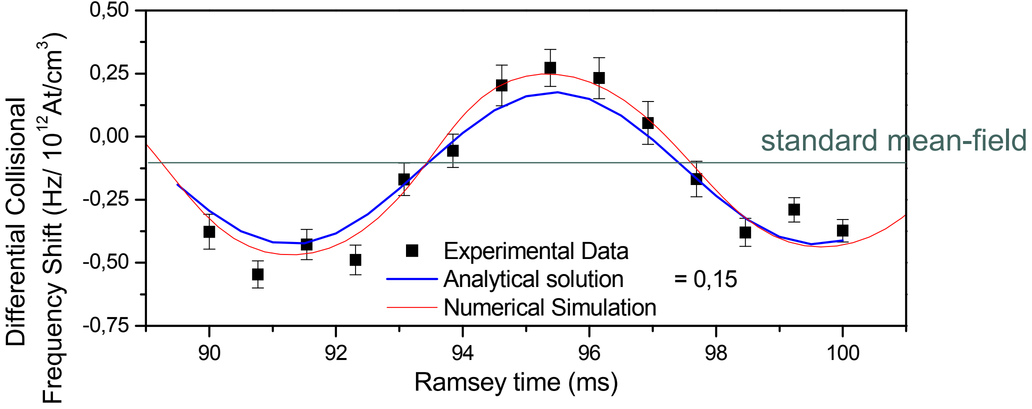
Optical Clock Fermion Collisions - Microwave Boson Clock Shift

- ^{87}Rb spin-exchange is almost mathematically identical to fermions.
- Dipolar spin waves.





$$\Delta\nu = -\frac{\sin(gT_R)}{T_R}\boxed{\frac{\cos(\omega_z T_R)}{4\pi}}\frac{\Delta\theta_1\Delta\theta_2\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

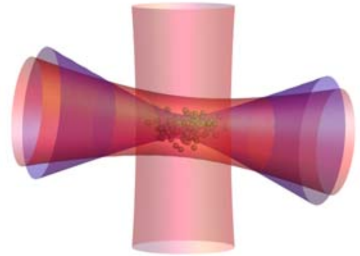


Maineult, Deutsch, KG, Reichel, Rosenbusch PRL ‘12

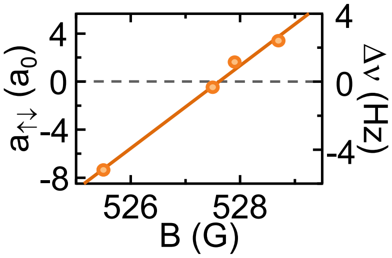
Observation of Fermion Clock Shifts

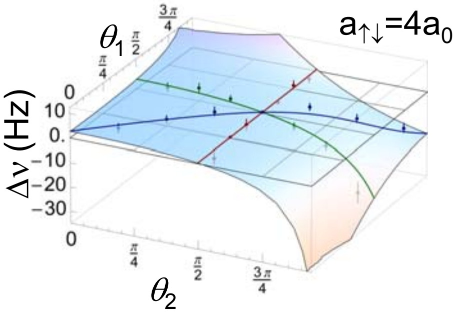
- Unique behaviors of Fermion shift, as compared to bosons (Fermion p-waves):

  1. Shift $\Delta\nu$ is independent of 1st Ramsey pulse area θ_1 ($n_\uparrow - n_\downarrow$).
  2. Depends strongly on 2nd Ramsey pulse θ_2.
     not observed for ^{87}Sr & ^{171}Yb → p-wave
  3. Proportional to s-wave scattering length $a_{\uparrow\downarrow}$.
  4. Increases with inhomogeneity as $\Delta\theta^2$.



^{6}Li s-wave
Feshbach
resonance



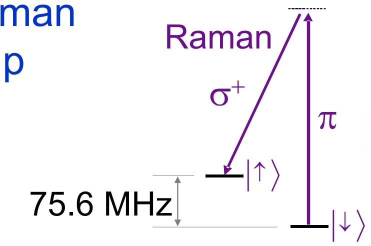


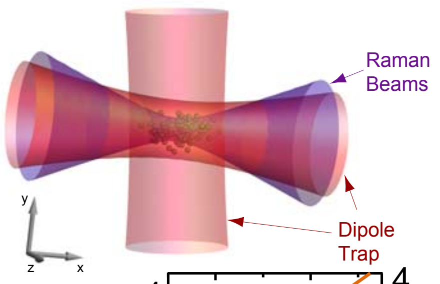
Campbell … Ye, Science ‘09
Lemke, … Oates, PRL ‘09
KG, PRL ‘09

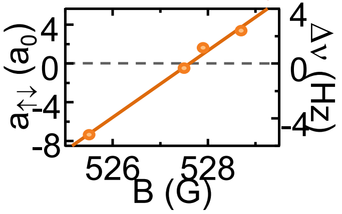
Maineult, Deutsch, KG, Reichel, Rosenbusch, PRL ‘12
Hazlett, Zhang, Stites, KG, O’Hara PRL ‘13

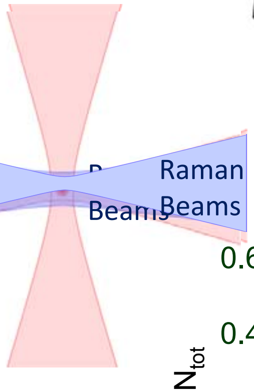
^{6}Li Fermion "Clock"

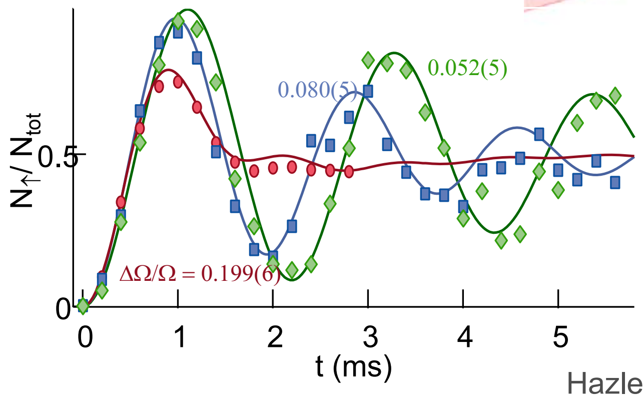
- Inhomogeneity from Raman beams – nuclear spin flip
- Resolved sideband: $\nu_{x,y,z}$={3.4, 7, 11} kHz
- Neglect trap-state changing collisions: $\lambda_{dB} \gg a_{\uparrow\downarrow}$

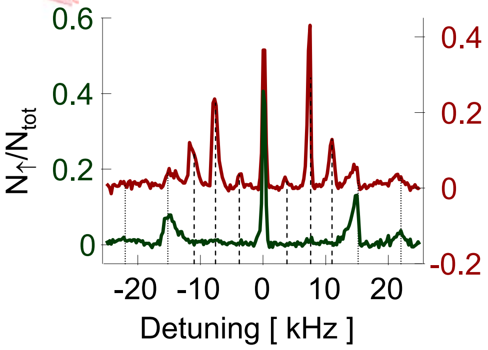








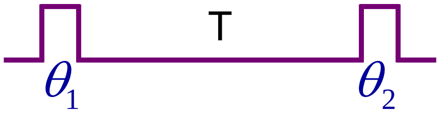




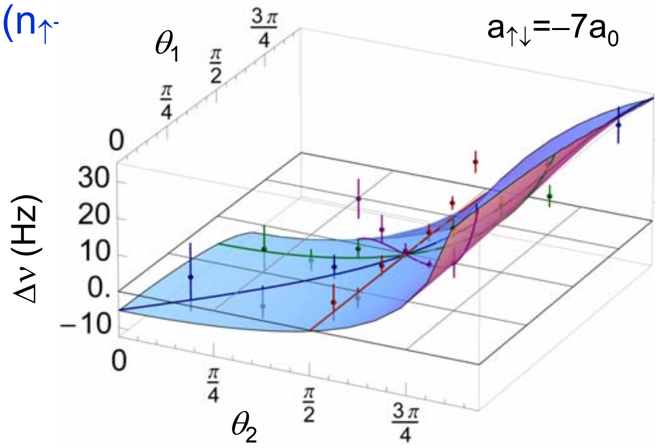
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

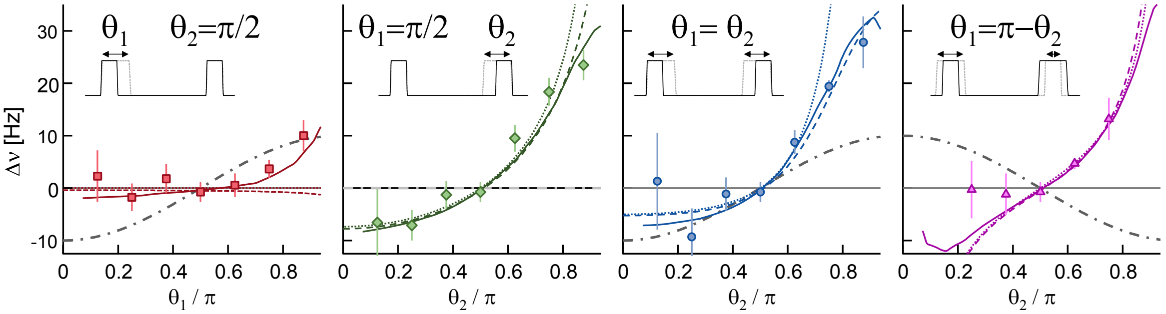
Fermion Clock Collision Shift

1. Shift $\Delta\nu$ is independent of θ_1 ($n_{\uparrow}$
2. Depends strongly on θ_2.
3. Proportional to $a_{\uparrow\downarrow}$.
4. Increases as $\Delta\theta^2$ (next).



$$\Delta\nu = \frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

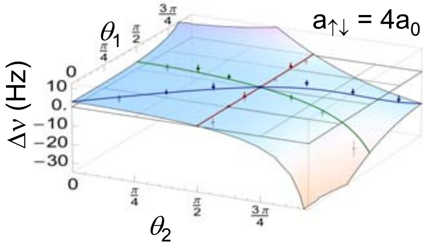




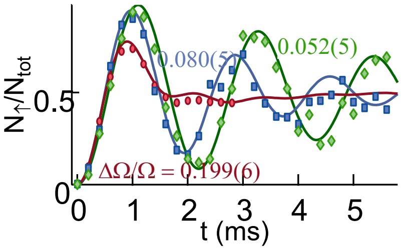
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

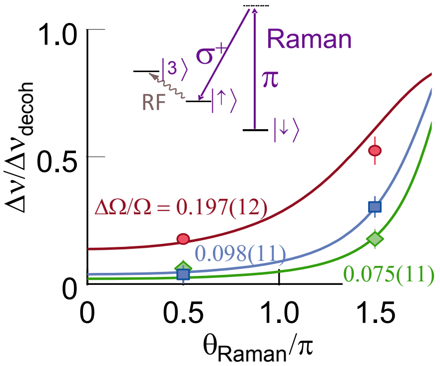
Measuring ΔΩ

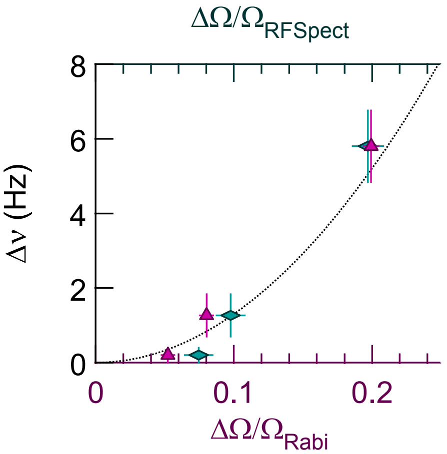
1. Shift $\Delta\nu$ is independent of θ_1 ($n_\uparrow - n_\downarrow$).
2. Depends strongly on θ_2.
3. Proportional to $a_{\uparrow\downarrow}$.
4. Increases as $\Delta\theta^2$.



Measure $\Delta\theta^2$ by Rabi flopping and frequency shift of ↑ to 3 transition.



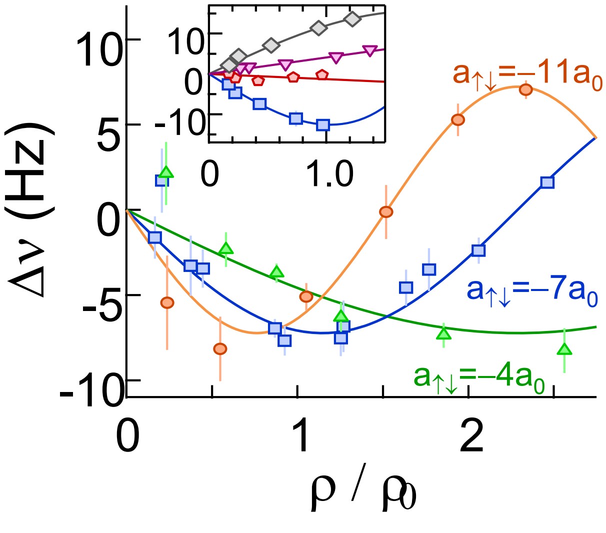




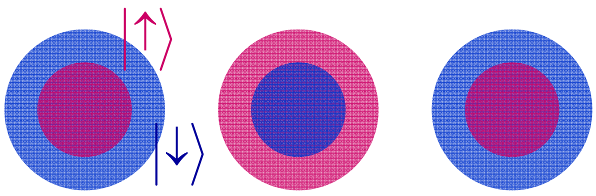
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

Spin Waves in Resolved Sideband Regime

- Fermion clock shift ⇔ spin waves
    - Low vibrational states have high Ω
    - Phase of singlet state evolves through π, and 2π.



$$\Psi = s e^{-i\omega_{ex}T}|0,0\rangle\{|\psi_a\psi_b\rangle\}^{+}$$
$$+\left[t|1,0\rangle + u|1,1\rangle + d|1,-1\rangle\right]\{|\psi_a\psi_b\rangle\}^{-}$$
$$=\tfrac{1}{\sqrt{2}}\left(t + s e^{-i\omega_{ex}T}\right)\{|\uparrow\psi_a\rangle|\downarrow\psi_b\rangle\}^{-}$$
$$+\tfrac{1}{\sqrt{2}}\left(t - s e^{-i\omega_{ex}T}\right)\{|\downarrow\psi_a\rangle|\uparrow\psi_b\rangle\}^{-}$$
$$+\left[u|\uparrow\uparrow\rangle + d|\downarrow\downarrow\rangle\right]\{|\psi_a\psi_b\rangle\}^{-}$$



Maineult, Deutsch, KG, Reichel, Rosenbusch, PRL '12

Hazlett, Zhang, Stites, KG, O'Hara PRL '13

Other Treatments

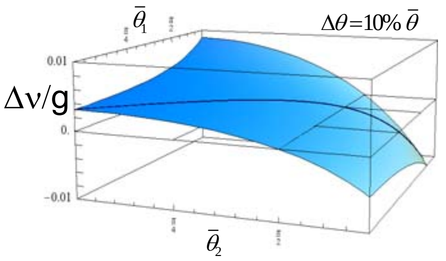
- Experimental papers modeling fermions as $h\Delta\nu=\mu_P-\mu_S$:

  Campbell, … Ye, Science ‘09   Lemke, … Oates, PRL ‘09   Blatt, … Ye, PRA ‘09

- Theoretical treatments:

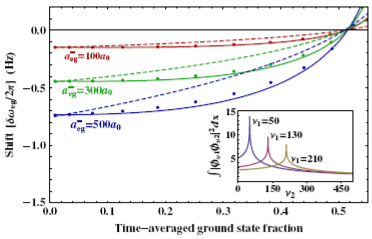
1. All agree on collision shift for Rabi excitation
2. Physical explanations and viewpoints are different – Ramsey

KG, PRL ‘09



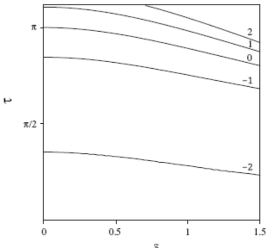
Ramsey & Rabi –
Fermions & Bosons
Singlet-triplet picture
Not proportional to
n_S-n_P.

Rey, Gorshkov, & Rubbo,
PRL ‘09



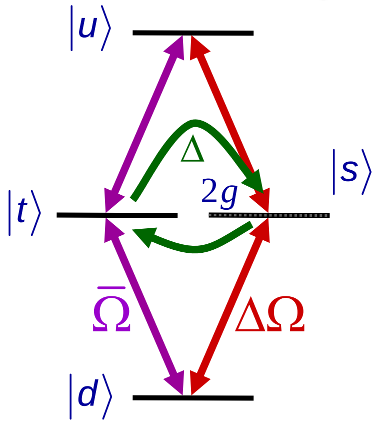
Rabi for Fermions
Consistent with
n_S-n_P &
$h\Delta\nu=\mu_P-\mu_S$

Yu & Pethik, PRL ‘10

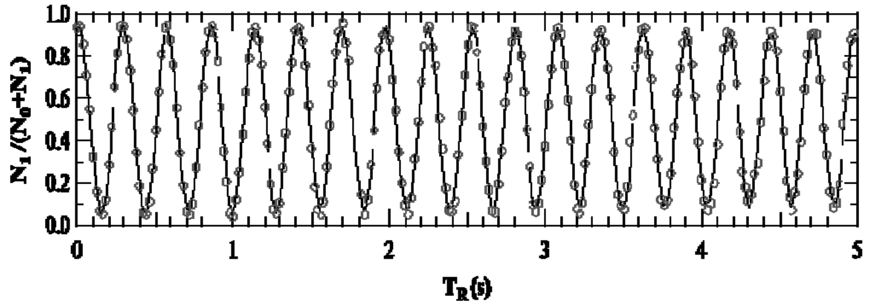


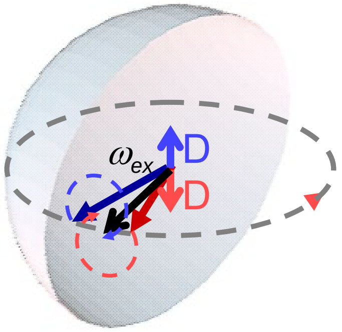
Rabi for Fermions
Spin precession
Consistent with
n_S-n_P?

Strong Interactions – More Coherence



- During Ramsey time, dephasing is a coupling Δ between s & t.
- Single state is detuned, $g>\Delta$
- Dephasing becomes off-resonant Rabi flopping
  - Self rephasing – spin echo?
- Contrast revivals occur when singlet gets $2n\pi$ collisional phase shift.
- Larger g for tighter traps, including tubes vs. pancakes

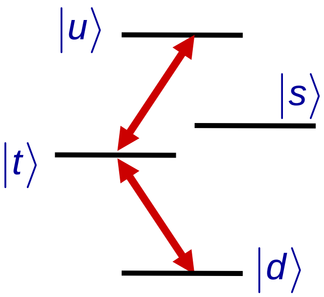


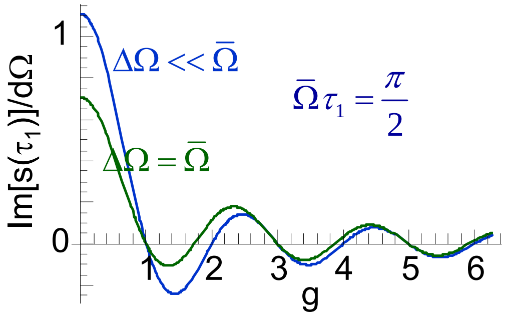


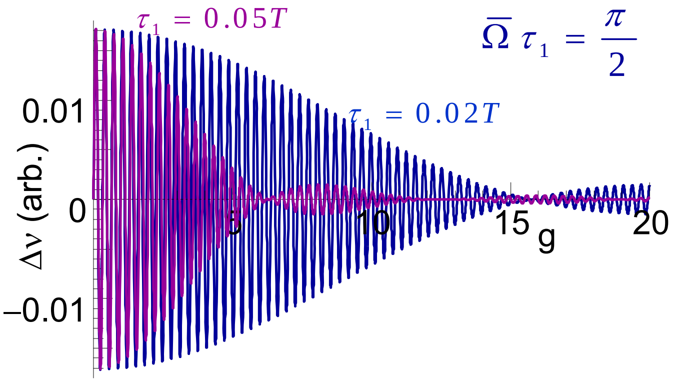
Deutsch, Ramirez-Martinez, Lacroûte … LaLoe, Reichel, Rosenbusch, PRL ’10
Gibble, Physics ‘10
Kleine Büning, Will, Ertmer, Rasel, Arlt, Klempt, … Rosenbusch, PRL ’11

Strong Interactions Eliminate Shift

- Single state is detuned, g>Ω
  - Long coherence times for Rb chip traps
- Shorter (1st) Ramsey pulse will excite singlet, longer pulses won't.
  - Eliminate shift by "resolving" singlet state.
- Random Ramsey phase (gT >>1)
  - gT >>1 alone doesn't give strong interactions
  - Distribution of g's ⇒ no shift!
  - How do you prove it?





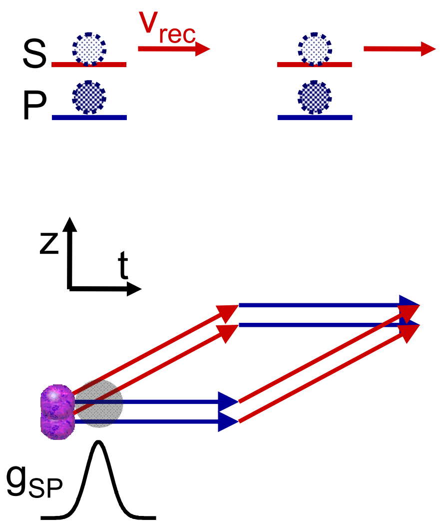


KG, Freq Contr. Symp. '10

Discussed in Swallows, … Ye, Rey, Science '11, but experiment was Sr p-wave.

Interferometers - "^{87}Rb is a boson"

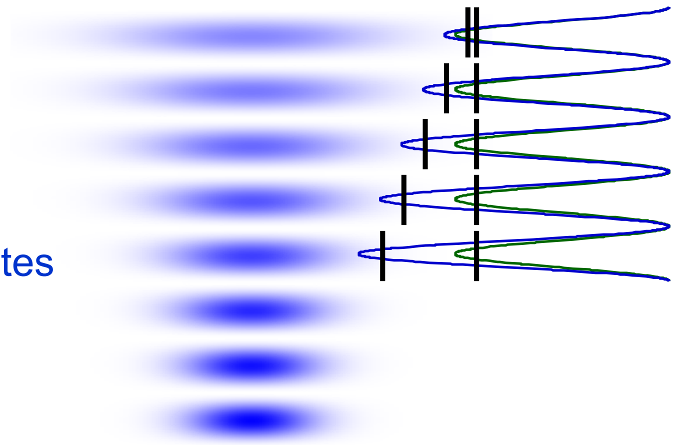
- Fermions or bosons for interferometers?
- Can particles be identical when their internal states have different momenta?
- What's the collision shift?
  - Resolved beam splitter diffraction
  - Tight radial confinement - wavepacket in z
  - Singlet & Triplet basis
- Fermions only interact for a short time
  - S & P interact for the entire time in a clock
- Bosons
  - SS and PP interact the entire time
  - S & P only for a short time



Inhibiting (single particle) Tunneling in μg

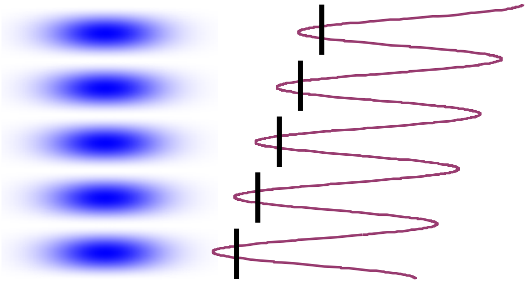
- Resonant tunneling in μg

1. Accelerated lattice
2. Simpler - don't use lattice waist region.
   - 0.1g is easy for 100 sites



- Tunneling is not resonant in accelerated lattices
  - Vertical lattice on earth

$mg(½\lambda) = 0.06\ E_r$

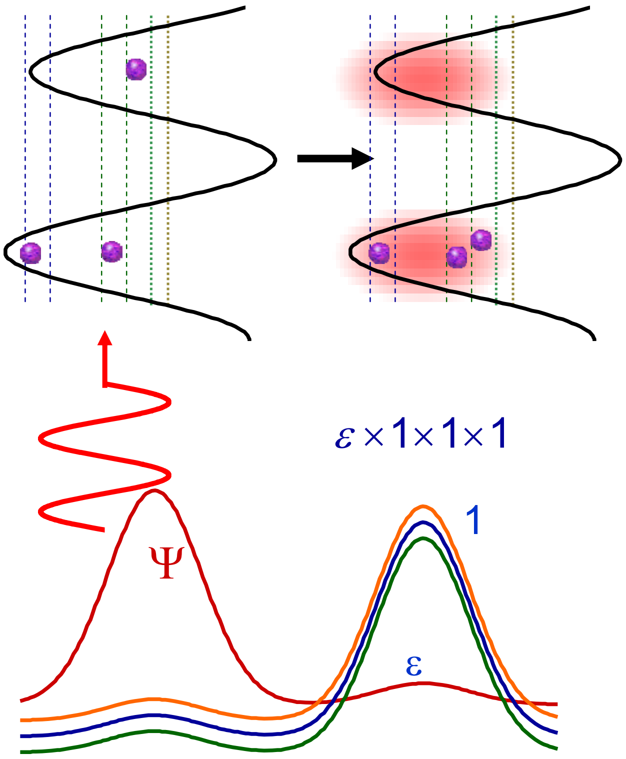


Lemonde & Wolf, PRA '05

Scattering Induced Tunneling

- Atoms in adjacent lattice sites have phase shift of ~π.
- After 1st scattering, could an atom produce a large frequency shift?
- Tunneling shift could be $\propto n^2$.
  - need a minimum of 3 particles

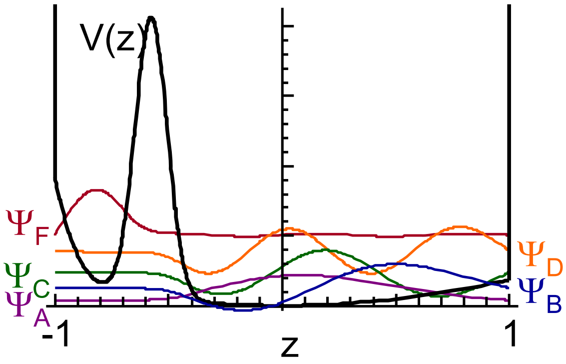
Scattering amplitude is not that small. Is shift ε or ε^2?

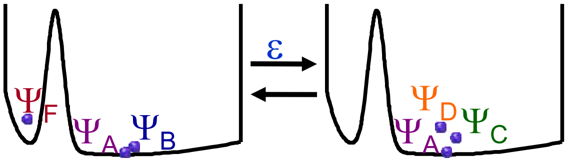


Gibble, SPIE '10

Scattering Induced Tunneling

- Consider a double well where only 1 scattering process is allowed.
- Classical picture – atom tunnels and then scatters a lot?
- Quantum: interference of scattered and unscattered.





$$\Psi_A\Psi_B\Psi_F \Leftrightarrow \Psi_A\Psi_C\Psi_D$$

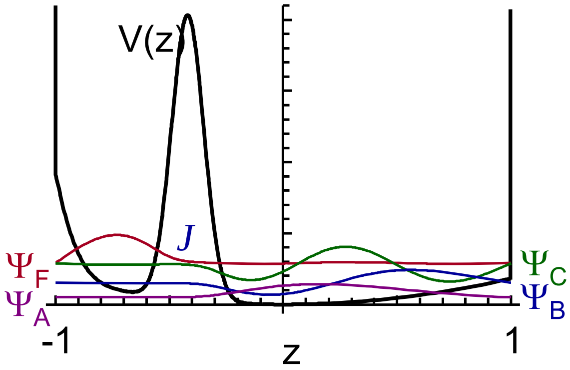
$$\Delta\nu = \sin^2\left(\frac{\phi_k}{2}\right)\frac{\sin(\theta_2)}{\pi A_c}\Big\{4T\varepsilon_{AF}\varepsilon_{BF}\sin(\phi_k)\sin^3(\theta_1)$$

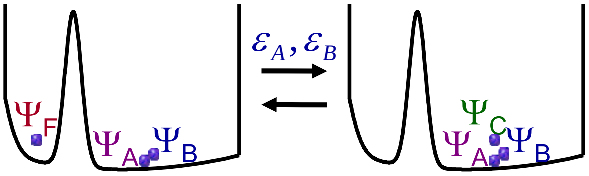
$$+\left[2(\varepsilon_{AF}+\varepsilon_{BF})-4\frac{T^2}{3}\left((g_{AC}+g_{AD}+4g_{CD})\boxed{\varepsilon^2}+g_{AB}(\varepsilon_{AF}-\varepsilon_{BF})^2\right)\right]\sin(2\theta_1)\Big\}$$

Only ε^2 – no interference for amplitude ε.

Scattering Induced Tunneling

- What if only the tunneling atom changes state?
- Then is there interference and a shift proportional to ε?





$$\Psi_A\Psi_B\Psi_F \Leftrightarrow \Psi_A\Psi_B\Psi_C$$

$$\Delta\nu = \frac{\sin(\theta_2)}{\pi A_c}\Big\{2J^2T\sin(\phi_k)\sin(\theta_1)+4T\varepsilon_{AF}\varepsilon_{BF}\sin^2\left(\frac{\phi_k}{2}\right)\sin(\phi_k)\sin^3(\theta_1)$$

$$+\Big[2(\varepsilon_{AF}+\varepsilon_{BF})-\frac{T^2}{3}\Big(g_{AC}(J+2\boxed{\varepsilon_A}+\boxed{\varepsilon_B})^2+g_{BC}(J+\boxed{\varepsilon_A}+2\boxed{\varepsilon_B})^2$$

Only ε^2.

$$+4g_{AB}\left[(\boxed{\varepsilon_A}-\boxed{\varepsilon_B})^2+(\varepsilon_{AF}-\varepsilon_{BF})^2\right]\Big)\Big]\sin^2\left(\frac{\phi_k}{2}\right)\sin(2\theta_1)\Big\}$$

Scattering induced tunneling gives a quadratic density dependence.

Order of magnitude is same as for single-particle tunneling.

Summary

- First observation of an ultracold collisional frequency shift of a Fermi gas.
  1. Shift $\Delta\nu$ is independent of θ_1 ($n_\uparrow - n_\downarrow$).
  2. Depends strongly θ_2.
  3. Increases with inhomogeneity as $\Delta\theta^2$.
- Applicable to fermion lattice clocks
  - Can often have smaller $\Delta\theta$.
- No trap-state changing collisions & resolved sidebands $\rightarrow$ observed predicted spin waves
- Strong Interactions improve coherence & eliminate shift.
- Correlations shift $\Delta\nu=0$ to $\theta_2=0.51\pi$
  - Max Ramsey fringe contrast biases to colder atoms, giving $\cos(\theta_2)=0$ at $\theta_2=0.56\pi$.
  - g correlated with $\theta_2 \rightarrow$ lower θ_2
  - g anti-correlated $\Delta\theta \rightarrow$ higher θ_2

$$\Delta\nu = \sum_{pairs} \frac{g}{2\pi} \frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$