Block I:

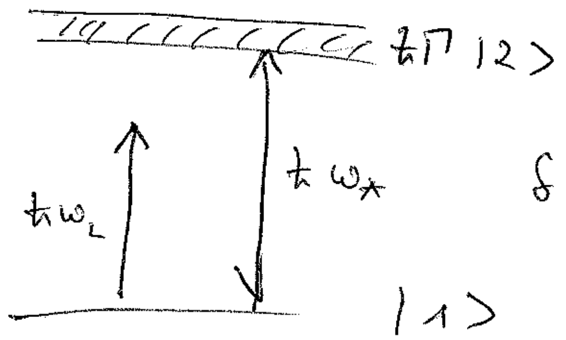
Basics of cooling, trapping and atom optics

1. Laser cooling and trapping of neutral particles

1.1. Light forces on atoms

× Average force on an atom placed in a near-resonant monochromatic laser beam

× Atom



$\delta = \omega_L - \omega_A \qquad |\delta| \ll \omega_L, \omega_A$

× Laser beam

$$\vec{E}_L(\vec{r},t) = \frac{1}{2}\, \mathcal{E}_L(\vec{r}) \left(e^{-i\omega_L t - i\Phi(\vec{r})} + c.c. \right)$$

$\Phi(\vec{r}) = -i\vec{k}_L \cdot \vec{r}$ plane wave

× Interaction between atom and laser in electric dipole approx.

typ. size of atom $\ll$ laser wavelength

$$V_{Int\,AL} = -\hat{\vec{D}} \cdot \vec{E}_L(\vec{R},t)$$

$\vec{R}$ center of mass position of the atom

$\hat{\vec{D}}$ electric dipole operator

$$\hat{\vec{D}} = \vec{d}\, |2\rangle\langle 1| + h.c.$$

$$\vec{d} = -e\, \langle 2|\vec{r}|1\rangle = -e \int d^3r'\, \varphi_2^*(\vec{r}\,')\, \vec{r}\,'\, \varphi_1(\vec{r}\,')$$

Rotating wave approximation (neglecting fast oscillating terms) e.g. $|1\rangle\langle 2|\, e^{+i\omega_L t} \simeq e^{+i(\omega_L+\omega_A)t}$ (2)

$$V_{IntAL} \simeq \frac{\hbar\,\Omega_1(\vec{R})}{2}\left[\,|2\rangle\langle 1|\,e^{-i\omega_L t - i\Phi(\vec{R})} + |1\rangle\langle 2|\,e^{i\omega_L t + i\Phi(\vec{R})}\right]$$

Absorption

stimulated emission

Rabi frequency

$$\hbar\,\Omega_1(\vec{R}) = -\vec{d}\cdot\vec{\mathcal{E}}_L(\vec{R})$$

Note $\Omega_1^2 \sim \mathcal{E}_L^2 \sim I$

Force exerted by the laser on the atom

Full Hamiltonian

$$\hat{H} = \frac{\hat{\vec{P}}^2}{2m} + \hbar\omega_A |2\rangle\langle 2| + \frac{\hbar}{2}\Omega_1(\vec{R}) \cdot \Big[|2\rangle\langle 1| e^{-i\omega_L t} e^{-i\Phi(\vec{R})} + |1\rangle\langle 2| e^{i\omega_L t} e^{i\Phi(\vec{R})} \Big]$$

In the Heisenberg picture, the force operator is defined by the following analogy to classical mechanics

$$\vec{F} = \frac{d\hat{\vec{P}}}{dt} = -\frac{i}{\hbar}\left[\hat{\vec{P}}, \hat{H}\right] = -\frac{i}{\hbar}\left[\hat{\vec{P}}, V_{Int}^{AL}\right]$$

Since $\hat{\vec{P}} = -i\hbar\nabla_{\vec{R}}$

$$\hat{\vec{F}} = -\nabla V_{Int}^{AL} = +\vec{\nabla}\left[\hat{\vec{D}} \cdot \vec{E}_L(\vec{R}, t)\right] = \sum_{i=x,y,z} \hat{D}_i \nabla E_L^{(i)}(\vec{R}, t)$$

We are interested in the expectation value for the force

$$\vec{F} = -\langle \nabla V_{Int}^{AL} \rangle = \Big\langle \sum_{i=x,y,z} \hat{D}_i \nabla E_L^{(i)}(\vec{R}, t) \Big\rangle$$

Since the spatial extension of the atom $\ll \lambda_L$

$$\langle E_L(\hat{\vec{R}}) \rangle = \vec{E}_L(\langle\hat{\vec{R}}\rangle) = \vec{E}_L(\vec{R}(t))$$

classical trajectory

(The atomic motion is being treated classically here, but the internal dynamics quantum mechanically)

$$\vec{F} = \sum_{i=x,y,z} \langle \hat{D}_i \rangle \nabla E_L^{(i)} (\vec{R}(t), t)$$

$$\cong \sum_{i=x,y,z} \langle D_i \rangle_{st} \; \nabla E_L (\vec{R}(t), t)$$

Not interested in fast dynamics of the atomic internal state, but only in stationary solution.

In particular, we will have to find the stationary value for the induced atomic dipole moment.

How to calculate?

→ From stationary solution of the optical Bloch equations.

Optical Bloch equations → time evolution of density matrix

$$i\hbar \dot{\hat{\rho}} = [\hat{\rho}, \hat{H}] \qquad \text{with } \hat{H} = -\frac{\hat{P}^2}{2m} + \hbar\omega_A |2\rangle\langle 2| + V_{int}^{LA}(t)$$

Reminder:

ρ_{22}, ρ_{11} population of levels $|2\rangle, |1\rangle$, respectively

$\rho_{21} = \rho_{12}^*$ coherences

Reminder:

Density matrix allows for the description of incoherent superpositions and statistical mixtures

Density matrix for a pure quantum mechanical state $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\rho = \begin{pmatrix} |c_1|^2 & c_1 c_2^* \\ c_2 c_1^* & |c_2|^2 \end{pmatrix}$$

Optical Bloch equations take into account spontaneous emission

$$\Longrightarrow \quad \dot{\rho}_{22} \qquad - \Gamma \rho_{22}$$

$$\dot{\rho}_{11} \qquad + \Gamma \rho_{11}$$

$$\dot{\rho}_{12} \qquad - \Gamma/2\, \rho_{12}$$

Coupled system of differential equations

$$\frac{d\rho_{22}}{dt} = -\Gamma\rho_{22} + \frac{i\Omega_1(\vec{R})}{2}\left(\rho_{21}e^{i\omega_L t + i\Phi(R)} - \rho_{12}e^{-i\omega_L t - i\Phi(R)}\right)$$

$$\frac{d\rho_{21}}{dt} = -\left(i\omega_A + \Gamma/2\right)\rho_{21} + \frac{i\Omega_1(R)}{2}(\rho_{22}-\rho_{11})e^{-i\omega_L t - i\Phi(R)}$$

$$\rho_{11} + \rho_{22} = 1 \qquad \rho_{12} = \rho_{21}^{*}$$

Introduce real quantities

u, v, w

$$u(t) = \frac{1}{2}\left(\rho_{12}\,e^{-i\omega_L t - i\Phi(\vec{R})} + \rho_{21}\,e^{i\omega_L t + i\Phi(\vec{R})}\right)$$

$$v(t) = \frac{1}{2i}\left(\rho_{12}\,e^{-i\omega_L t - i\Phi(\vec{R})} - \rho_{21}\,e^{i\omega_L t + i\Phi(\vec{R})}\right)$$

$$w(t) = \frac{1}{2}(\rho_{22} - \rho_{11})$$

$$\Longrightarrow \quad \dot{u} = -\frac{\Gamma}{2}u + \delta v$$

$$\dot{v} = -\delta u - \frac{\Gamma}{2}v - \Omega_1(\vec{R})\,w$$

$$\dot{w} = \Omega_1(\vec{R})\,v - \Gamma w - \frac{\Gamma}{2}$$

$$u_{st} = \frac{\delta}{\Omega_1(\vec{R})}\,\frac{s(\vec{R})}{1+s(\vec{R})}$$

$$v_{st} = \frac{\Gamma}{2\Omega_1(\vec{R})}\,\frac{s(\vec{R})}{1+s(\vec{R})}$$

$$w_{st} = -\frac{1}{2}\,\frac{1}{1+s(\vec{R})} \qquad s(\vec{R}) = \frac{\Omega_1^2(\vec{R})/2}{\delta^2 + \Gamma^2/4}$$

How to calculate $\langle \hat{\vec{D}} \rangle$ with ρ_{ij}?

$$\langle \hat{\vec{D}} \rangle = \mathrm{Tr}\left\{ \hat{\rho}\, \hat{\vec{D}} \right\}$$

$$= \vec{d}\ \mathrm{Tr}\left\{ \hat{\rho} \left(|2\rangle\langle 1| + |1\rangle\langle 2| \right) \right\}$$

$$= \vec{d} \left(\rho_{21,st} + \rho_{12,st} \right)$$

Now, let us assume

$$E_L(\vec{R},t) = \mathcal{E}_L(\vec{R}) \cos\left(\omega_L t + \Phi(\vec{R})\right)$$

$$\Rightarrow \langle \hat{\vec{D}} \rangle_{st} = 2\vec{d} \Big[\left(u_{st} \cos(\omega_L t + \Phi(\vec{R})) \right.$$
$$\left. - v_{st} \sin(\omega_L t + \Phi(\vec{R})) \right) \Big]$$

in phase component

out of phase component

Calculate the force:

$$\vec{F} = -2\hbar \left(u_{st} \cos(\omega_L t + \Phi(\vec{R})) - v_{st} \sin(\omega_L t + \Phi(\vec{R})) \right)$$
$$\cdot \nabla \left(\Omega_1(\vec{R}) \cos(\omega_L t + \Phi(\vec{R})) \right)$$

$$= -2\hbar \Big(\qquad \Big)$$
$$\cdot \Big[\nabla \Omega_1(\vec{R}) \cos(\omega_L t + \Phi(\vec{R}))$$
$$- \Omega_1(\vec{R}) \nabla\Phi(\vec{R}) \sin(\omega_L t + \Phi(\vec{R})) \Big]$$

$$\overline{\vec{F}}_{\text{time av}} = -2\hbar \nabla \Omega_1(\vec{R}) \cdot \frac{1}{2} u_{st} - 2\hbar v_{st} \Omega_1(\vec{R}) \nabla \Phi(\vec{R}) \cdot \frac{1}{2}$$

$$= -\hbar \Omega_1(\vec{R}) \left[\frac{\nabla \Omega_1(\vec{R})}{\Omega_1(\vec{R})} u_{st} + v_{st} \nabla \Phi(\vec{R}) \right]$$

$$= -\hbar \Omega_1(\vec{R}) \left[\frac{\nabla \Omega_1(\vec{R})}{\Omega_1(\vec{R})} \delta + \frac{\Gamma}{2} \nabla \Phi(\vec{R}) \right] \frac{s(\vec{R})}{1+s(\vec{R})}$$

Two forces

(1) $\sim P\Omega_1$ $\sim$ gradient of intensity

(in phase component)

Dipole force

(2) $\sim P\Phi$ $\sim$ gradient of phase

$\rightsquigarrow$ dissipative force

$\rightarrow$ radiation pressure

1.2. Radiation pressure

Assume: $\Phi(\vec{R}) = -\vec{k}_L \cdot \vec{R}$

$$\Longrightarrow \vec{F}_{RP} = \underbrace{\hbar \vec{k}_L}_{\text{photon momentum}} \underbrace{\frac{\Gamma}{2} \frac{s(\vec{R})}{1+s(\vec{R})}}_{\text{Scattering rate of photons on a closed transition in a two-level system}}$$

Force has Lorentzian profile as a function of δ with a total width of $(\Gamma^2 + 2\Omega_1^2)^{1/2}$

$$\vec{F}_{RP} \xrightarrow[s \to \infty]{} \hbar \vec{k}_L \frac{\Gamma}{2}$$

1.3. Using radiation pressure to slow down the motion of atoms

Atom ○ $\longrightarrow$ $\quad\quad$ $\rightsquigarrow$ $\leftsquigarrow$ laser light

$$\vec{F} = \hbar \vec{k}\, \Gamma_{scatt}$$

$$= \hbar \vec{k}\, \frac{\Gamma}{2} \frac{s_0}{1+s}$$

Consider Doppler effect. To be on resonance, you have to detune the laser to be on resonance with the moving atom that means

$$\omega_L = \omega_A + \vec{k}\vec{v}$$

Estimate of max. force / max. deceleration

$$a_{max} = \frac{F_{max}}{m} = \frac{\hbar k}{m} \frac{\Gamma}{2} = v_r \frac{\Gamma}{2}$$

recoil velocity

Sodium: D_2 line:

$\lambda = 589\,nm$ $\quad\quad$ $k = \frac{2\pi}{\lambda}$

$m = 23\ amu$ $\quad\quad$ $\tau = 16\,ns$

$$\Longrightarrow \quad a_{max} = \frac{\hbar k}{2\tau m} \approx 10^5 g$$

Length scale for slowing down atoms

$$\frac{dv}{dt} = -a_{typ} = -\frac{a_{max}}{2}$$

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} = \frac{1}{2}\frac{dv^2}{ds} = -a_{typ}$$

$$v^2(s) = v_0^2 - 2a_{typ}s$$

Ask for $v^2(L) \stackrel{!}{=} 0$ at $s = L$

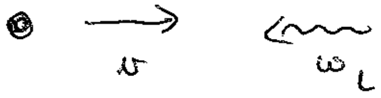
$$\Rightarrow L = \frac{v_0^2}{2a_{typ}}$$

Sodium: $T = 1000\,K$ $\quad v_0 \approx 1000\,\frac{m}{s}$

$$\Rightarrow L = 1{,}1\,m$$

Of course, this only holds when the laser remains on resonance with the atom during slow down.

Problem: Doppler shift



$$\omega_L^k = \omega_A + \vec{k}\cdot\vec{v}$$

$$\Delta \nu_D \,(@\, 1000\, m/s) = \frac{k v}{2\pi} \approx 2\,GHz \qquad \text{for Na}$$

$$\Delta \nu_{nat} = \Gamma_{sp} \approx 10\,MHz$$

$\Longrightarrow$ Without somehow adapting the frequency of the laser to the ~~at.~~ atom that is being slowed down, or without adapting the resonance frequency of the atom, you can only slow down the atom by $\Delta v = \frac{\hbar k}{m} = \frac{5\,m}{s}$ .....

$\Longrightarrow$ Chirp Cooling

- $\rightarrow$ Adapting laser frequency during slow down
- $\rightarrow$ pulsed atomic beams

or Zeeman-slower

1.4. Doppler ~~Laser~~ cooling of atomic ensembles in atomic molasses

Idea of temperature:

× Let us consider an atomic ensemble in free space without potential energy

× Thermal equilibrium

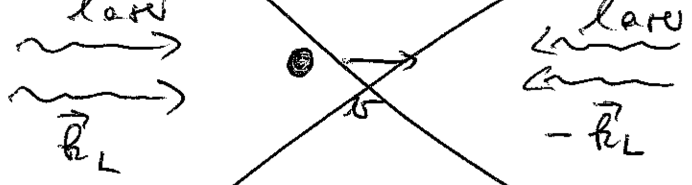
→ velocity distribution of the sample follows a Maxwell-Boltzmann distribution

$$f(v) = \frac{1}{\sqrt{2\pi}\,\tilde{v}(T)}\, e^{-v^2/2\tilde{v}(T)} \qquad \tilde{v}(T) = \sqrt{\frac{k_B T}{m}}$$

× Temperature of the ensemble ~~can~~ be read out from velocity spred in the system

× Cooling → reducing velocity spread!

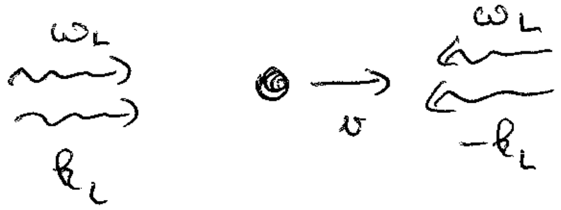
~~Mechanism of Doppler cooling~~



~~× Consider an atom moving along the x-axis interacting with two counterpropagating laser beams~~

Mechanism of Doppler cooling

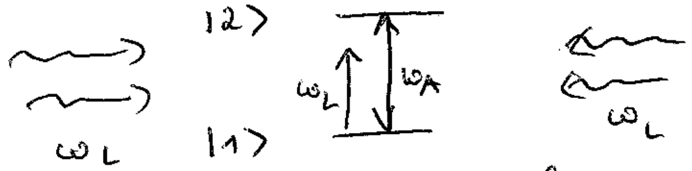
× Consider an atom moving along the x-axis interacting with two counter-propagating laser fields.



× Because of the Doppler effect

$F_{left} \neq F_{right} \rightarrow$ force

How can you use this force to cool atoms?



$$\omega_A + kv = \omega_L$$

× Choose $\omega_L < \omega_A$ red-detuned

→ The force of the counterpropagating laser field will always be larger than the force of the co-propagating laser field ⟹ force which ~~directs~~ ~~up~~. slows down the atomic motion

Quantitatively

$$F_{atom} = F_{eff} - F_{right}$$

$$= F_{scatt}(\omega_L - kv - \omega_A) - F_{scatt}(\omega_L + kv - \omega_A)$$

Assume $kv \ll \Gamma$ and develop the above expression in a Taylor expansion

$\Longrightarrow$

$$F_{atom} \approx F_{scatt}(\omega_L - \omega_A) + kv \frac{\partial F_{scatt}}{\partial \delta} \frac{\partial \delta}{\partial (kv)}$$

$$- F_{scatt}(\omega_L - \omega_A) - kv \frac{\partial F_{scatt}}{\partial \delta} \frac{\partial \delta}{\partial (kv)}$$

$$= -2kv \frac{\partial F_{scatt}}{\partial \delta}$$

$$F_{scatt}(\delta) = \hbar k \frac{\Gamma}{2} \frac{\Omega_1^2/2}{\frac{\Omega_1^2}{2} + \frac{\Gamma^2}{4} + \delta^2}$$

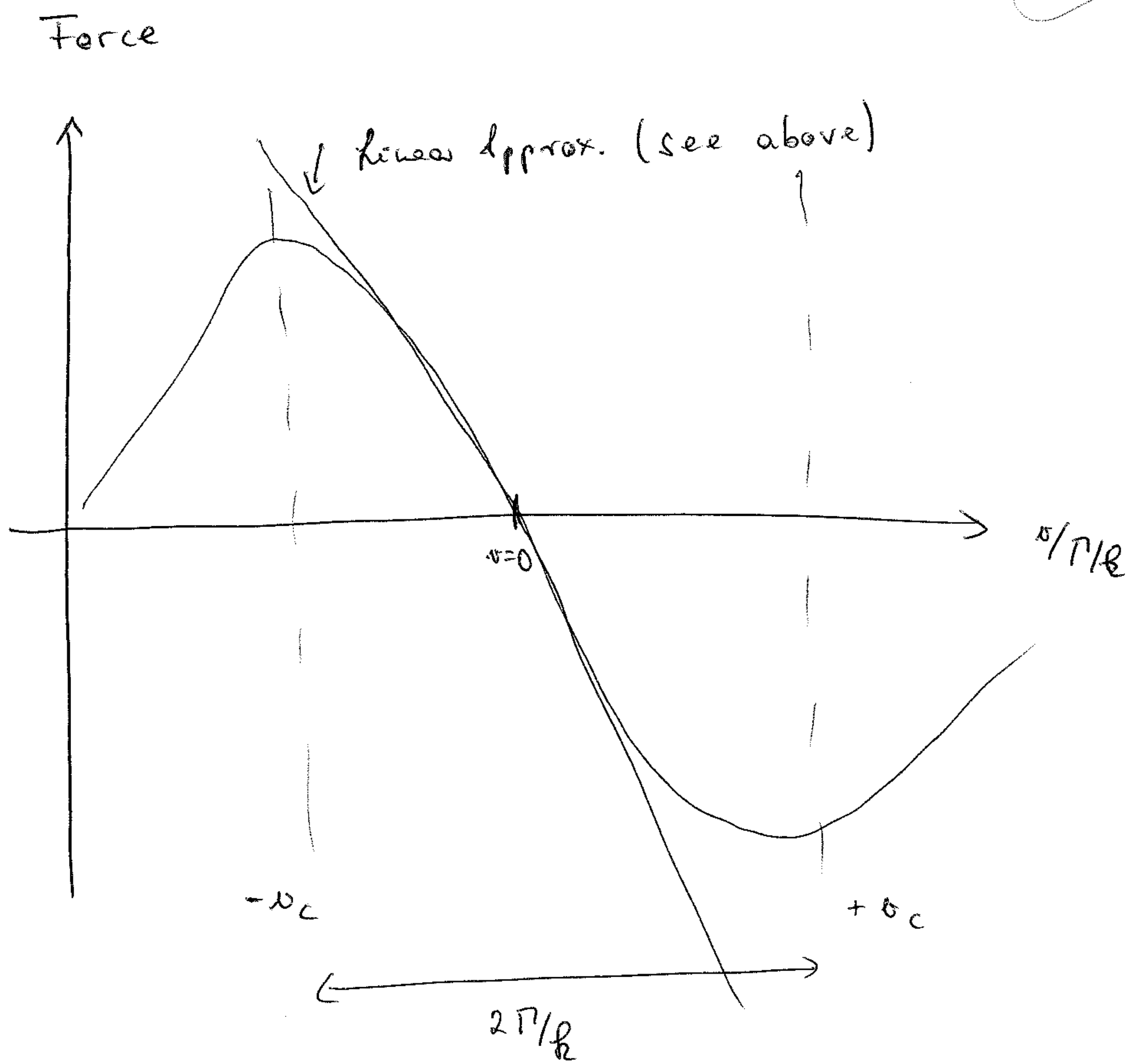
$$\frac{\partial F_{scatt}}{\partial \delta} = -\hbar k \frac{\Gamma}{2} \frac{\Omega_1^2/2}{\left[\frac{\Omega_1^2}{2} + \frac{\Gamma^2}{4} + \delta^2\right]^2} \cdot 2\delta$$

$$F_{atom} = +2kv\hbar k \frac{\Gamma}{2} \frac{\Omega_1^2/2}{\frac{\Omega_1^2}{2} + \frac{\Gamma^2}{4} + \delta^2} \cdot 2\delta$$

$$= \cancel{2}\hbar k^2 \Gamma \frac{\Omega_1^2/2}{\frac{\Omega_1^2}{2} + \Gamma^2/4 + \delta^2} \cdot \delta \cdot v$$

$$F_{atom} = -\alpha v \quad \text{mit} \quad \alpha = -2\hbar k^2 \Gamma \frac{\Omega_1^2/2}{\frac{\Omega_1^2}{2} + \Gamma^2/4 + \delta^2} \cdot$$

$\alpha >$ ~~Da~~ friction force for $\omega_L < \omega_A$



Critical velocity $v_c = \Gamma/k \approx 10 \frac{m}{s}$

How fast will the motion of the atom be cooled?

$$m\dot{v} = -|\alpha| v \longrightarrow v = e^{-|\alpha| t/m} v_0 = e^{-t/\tau} v_0$$

with $s \approx 1/10$ $\quad \delta = -\Gamma/2$

$$\Longrightarrow \underline{\tau \approx 100\,\mu s}$$

Limit to cooling!

× Because of spontaneous emission and the randomness of absorption
→ random walk

These fluctuations lead to a growth of the variance of the momentum with time: $\Delta p^2 = 2 D_p t$

D_p: diffusion coefficient

Contribution to D_p due to randomness of spontaneous emission.

Number of spontaneous emission events per interval

$$\Delta n = \Gamma \rho_{ee}^{st} \Delta t \qquad \rho_{ee}^{st} = \frac{1}{2} \frac{S}{1+S}$$

⇒ momentum spread

$$\Delta p^2 = (\hbar k_L)^2 \Delta n = 2 \left[\frac{\hbar^2 k_L^2 \Gamma}{2} \frac{S}{1+S} \right] \Delta t$$

$$D_p^{spont} = \frac{\hbar^2 k_L^2}{2} \Gamma \frac{S}{1+S} \overset{S \ll 1}{\simeq} \frac{\hbar^2 k_L^2 \Gamma}{2} S$$

Contribution from randomness of light absorption

$$D_p^{abs} \simeq \frac{\hbar^2 k_L^2 \Gamma}{2} S$$

Total

$$D_p \simeq \hbar^2 k_L^2 \Gamma_s$$

3D Molasses

$$D_p = 3\hbar^2 k_L^2 \Gamma_s$$

Cooling

$$\frac{dp^2}{dt} = 2p\frac{dp}{dt} = 2p\,m\frac{dv}{dt}$$

$$= -2p\cancel{m}|\alpha| v = -2|\alpha| p^2$$

$$\frac{dp^2}{dt} = \underbrace{-2|\alpha| p^2}_{\text{cooling}} + \underbrace{2D_p}_{\text{heating}} \overset{!}{=} 0$$

$$\left(\frac{p^2}{2m}\right)_{eq} = \frac{D_p}{|\alpha| 2m}$$

$$\frac{3}{2}k_B T = \frac{p^2}{2m} = \frac{D_p}{|\alpha| 2m}$$

$$k_B T = \frac{D_p}{3m|\alpha|} = \frac{\hbar\Gamma}{4}\left[\frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|}\right]$$

Minimal temperature

@ $\delta = -\Gamma/2$ $\qquad k_B T = \frac{\hbar \Gamma}{2}$

Doppler temperature

e.g. Rb $T_D \simeq 140 \mu K$

Na $T_D \simeq 240 \mu K$

H $T_D \simeq 2.4 mK$