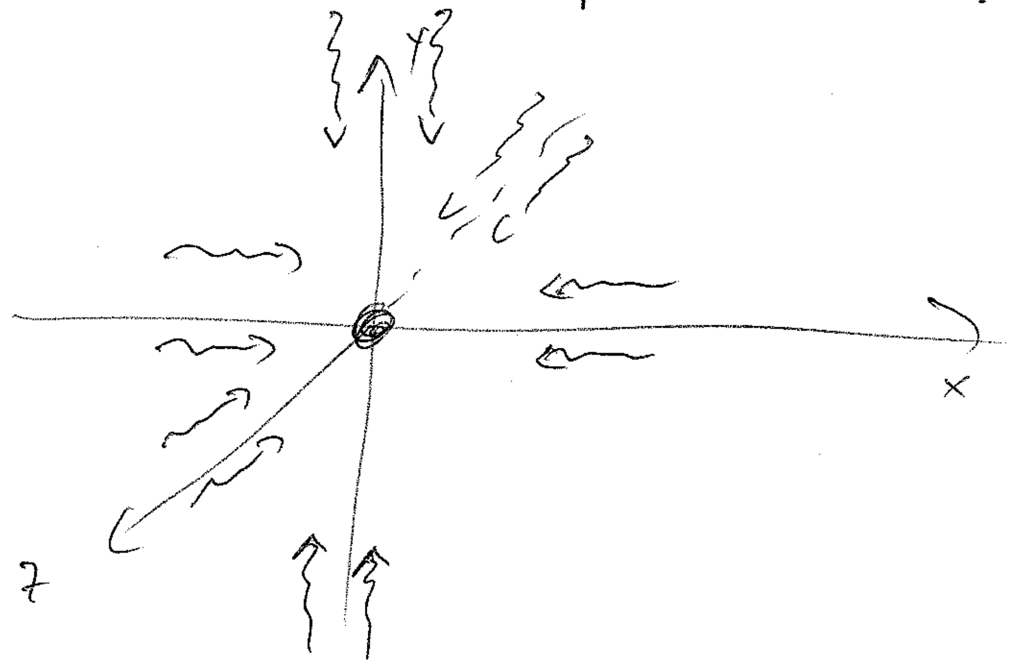
1.5 Laser trapping of atoms – the magneto-optical trap

In the previous lecture, we saw how to make use of light to cool atoms.
So far, however, the atoms will only be cooled and not be trapped.

Now: how to make use of radiation pressure to <u>trap</u> atoms

Configuration for optical molasses:



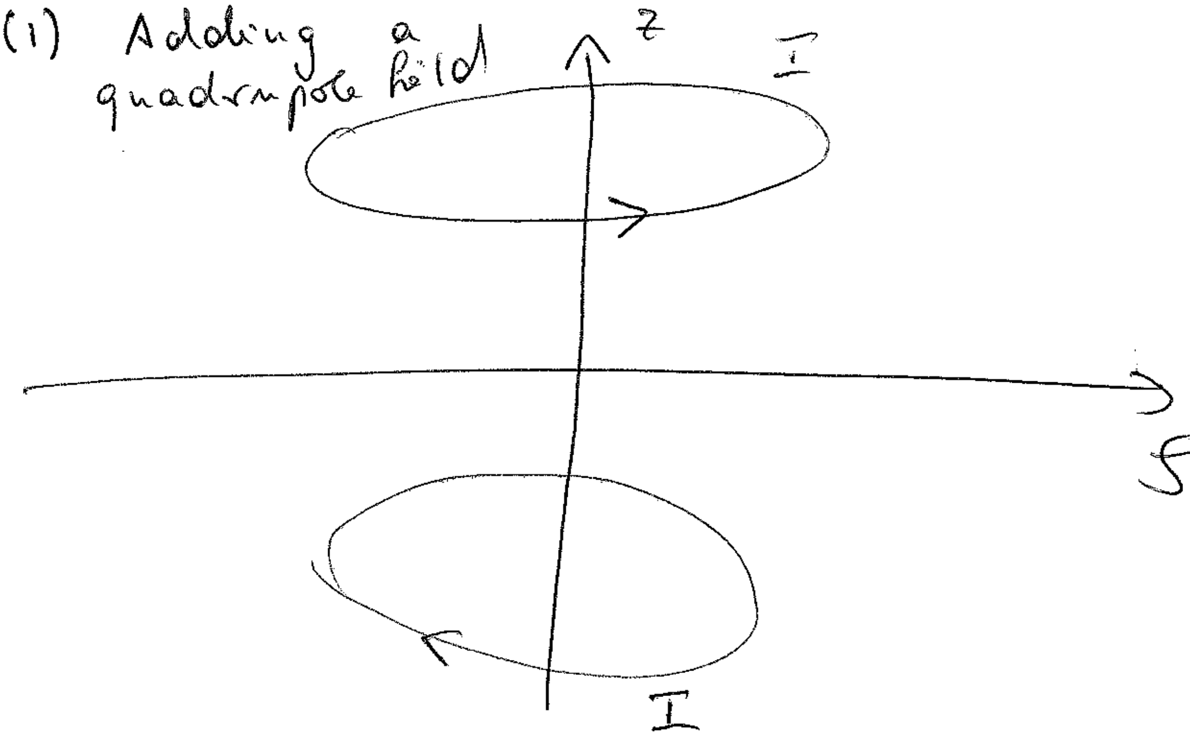
Atoms experience a friction force

$$\vec{F} = -\alpha \vec{v}$$

, but no trapping force, which would be $\sim \vec{x}$

Extend the optical molasses to a magneto-optical trap by ~~adding~~

(1) Adding a quadrupole field



Anti - Helmholtz - configuration

→ quadrupole field

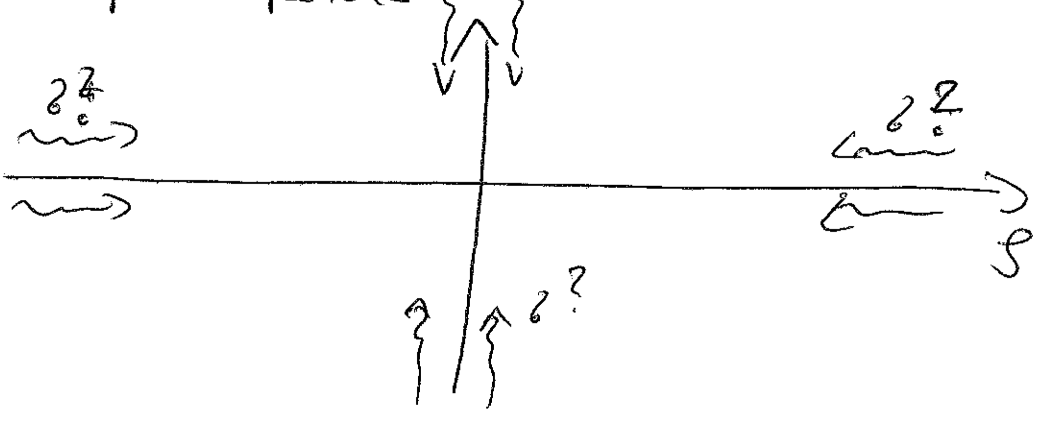
$$B(z,\rho) = A\sqrt{\rho^2 + 4z^2}$$

Close to the origin of the coordinate system

$$B(0,\rho) \approx A\rho$$

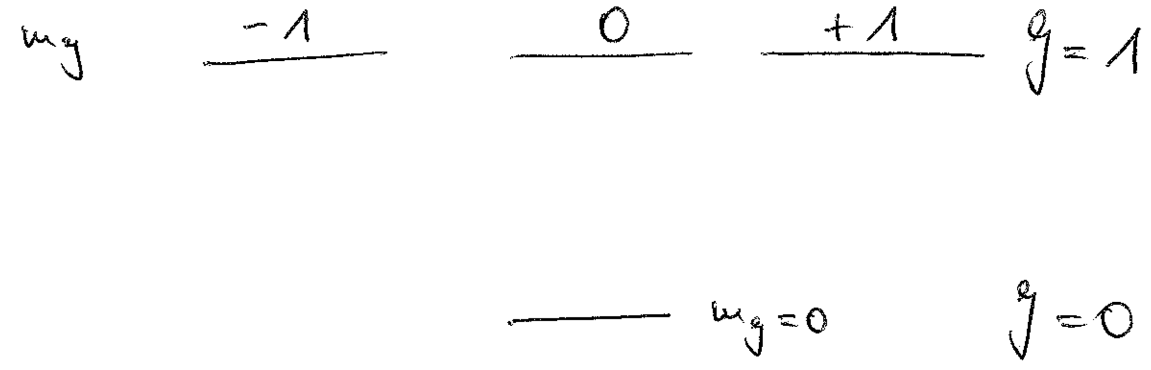
$$B(z,0) \approx 2Az$$

(2) ~~Now~~ Carefully choose polarization of the light fields σ?

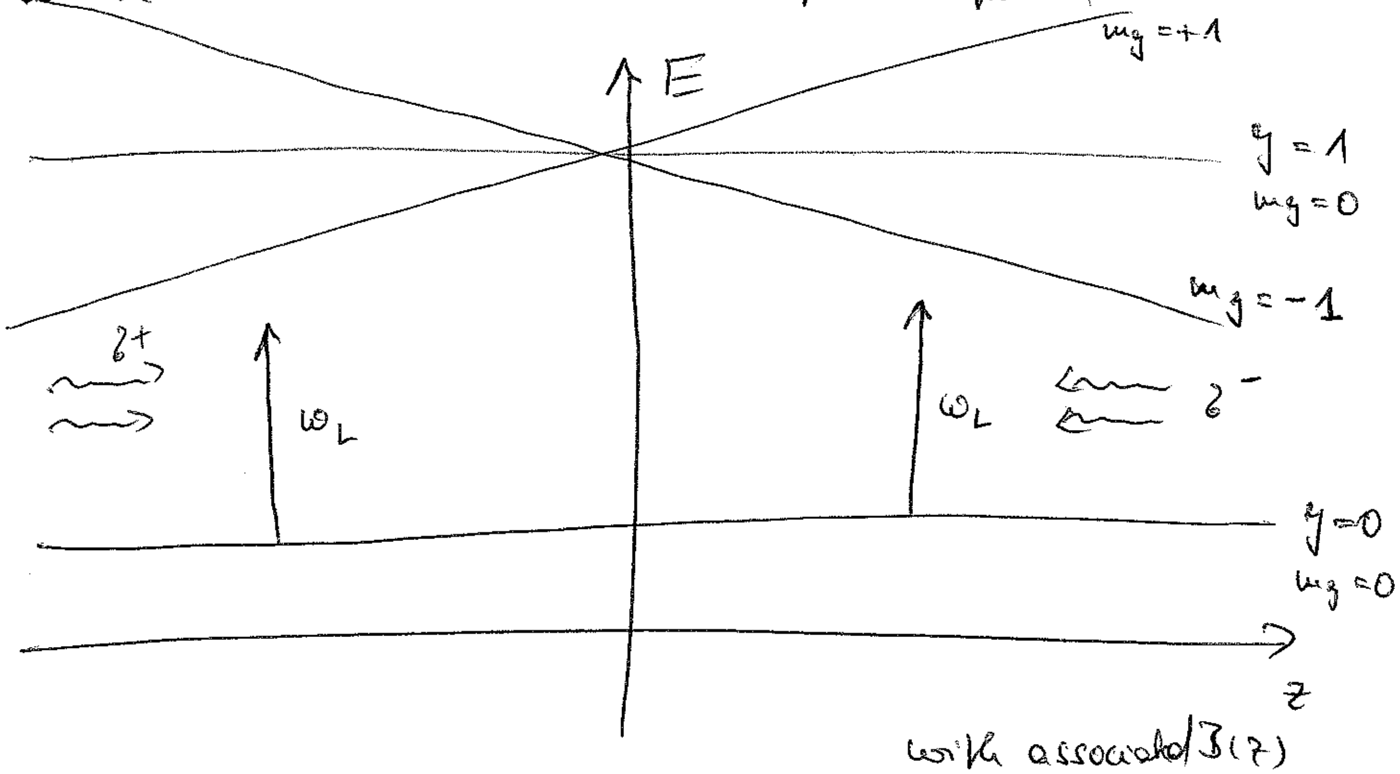


(3) Consider a multilevel atom

For the simplest possible model of a magneto-optical trap, choose the ground state of the atom to be a $j=0$ state and the excited state to be a $j=1$ state



Consider now the energy structure of this atom in an external magnetic field as above



with associated $B(z)$

- Now, red-detuned counter-propagating laser beams along z-axis.
- Choose the polarization as above σ^+/σ^-

× Consider how an atom ~~moving~~ in the (4)
$g=0$, $m_g=0$ state moving along the positive z-direction with velocity v

× In addition to the velocity dependent force that we derived in case of the optical molasses, we will get a position dependent force

Reason: σ^- light induces transitions

$m_g=0 \rightarrow m_g=-1$

It is therefore not so far detuned as σ^+ light ($m_g=0 \rightarrow m_g=+1$)

Atoms moving to the right at $z>0$ are "feeling" the σ^- laser much stronger than the σ^+ laser.

$\Longrightarrow$

$$F_{MOT} = F_{scatt}^{\sigma^+}\left[(\omega_L - kv) - (\omega_0 + \beta z)\right] - F_{scatt}^{\sigma^-}\left[\underbrace{(\omega_L + kv) - (\omega_0 - \beta z)}_{\delta}\right]$$

$$\simeq -2\frac{\partial F}{\partial \delta} kv - 2\frac{\partial F}{\partial \delta}\beta z \qquad \text{with } \beta z = \frac{g\mu_B}{\hbar}\frac{dB}{dz}z$$

$$F_{MOT} = \underset{\text{damping}}{-\alpha v} - \underbrace{\frac{\alpha\beta}{k} z}_{\text{restoring force}}$$

Damped harmonic oscillator

$$\Gamma_{MOT} = \frac{\alpha}{M} \qquad \omega_{MOT} = \sqrt{\frac{\kappa \beta}{k m}}$$

$$\left.\begin{array}{l} \omega_{MOT} \sim k\text{Hz} \\ \Gamma_{MOT} \sim 100\ k\text{Hz} \end{array}\right\} \text{overdamped}$$

Capture velocity:

5 mm laser beam diameter

$$5\,\text{mm} = L_0 = \frac{v_0^2}{a_{max}} \qquad \underline{\underline{v_c \approx 70\ \frac{m}{s}}}$$

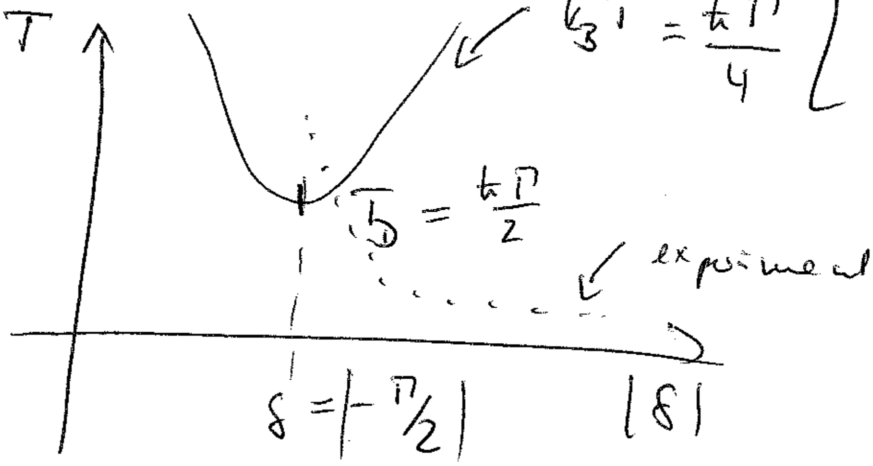
1.6. Sub-Doppler laser cooling mechanisms

1988: W. Phillips et al.
Observation of Atoms Laser Cooled below the Doppler limit
PRL 61, 169 (1988)

Experiment:

- Sodium atoms in an optical molasses with counterpropagating beams in 3D, lin. polarization

$$k_B T = \frac{\hbar\Gamma}{4}\left[\frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|}\right]$$



- Observation:
  With increasing detuning (red-detuning) δ, the temperature of the optical molasses decreases

- temperatures down to $T_D/5 - T_D/10$

Nobel price: 1997: Chu, Phillips, Cohen-Tannoudji

1.6.1. Sisyphus Cooling

× Consider the following molasses configuration

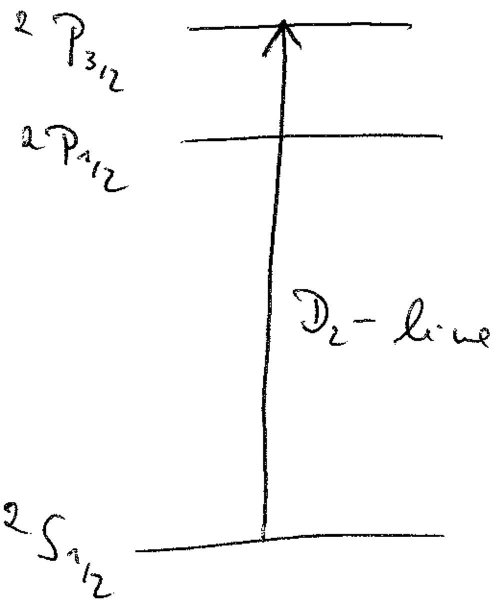
$\delta < 0$

× Consider an atom with the following term scheme

$J = 3/2$ $-3/2$ $-1/2$ $+1/2$ $+3/2$

$J = 1/2$ $-1/2$ $+1/2$

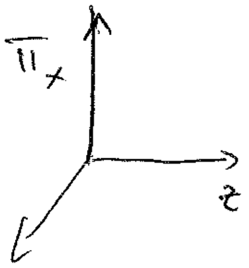
e.g. Alkali atoms

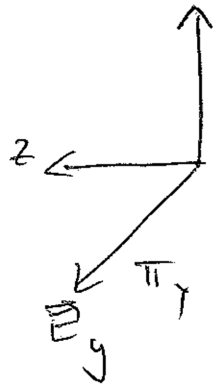


e

Choose the polarisation of the counter-propagating beams to be lin $\perp$ lin.

That means



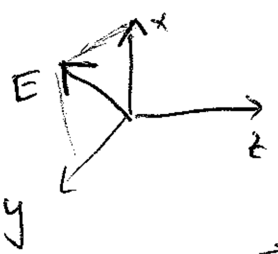


× polarisation of the beams orthogonal to the direction of propagation and polarization of the beams orthogonal to each other

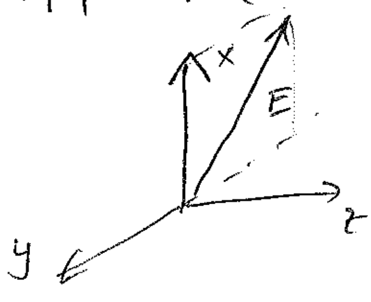
$$\vec{E} = E_0 \hat{x} \cos(\omega_L t - kz) + E_0 \hat{y} \cos(\omega_L t + kz)$$
$$= E_0 (\hat{x}+\hat{y}) \cos(\omega_L t) \cos(kz)$$
$$+ E_0 (\hat{x}-\hat{y}) \sin(\omega_L t) \sin(kz)$$

@ $z=0$

$$\vec{E}(z=0) = E_0 (\hat{x}+\hat{y}) \cos(\omega_L t)$$



@ $z=\lambda/4$ $\quad \vec{E}(z=\lambda/4) = E_0(\hat{x}-\hat{y}) \sin(\omega_L t)$



@ $z = \frac{\lambda}{8}$

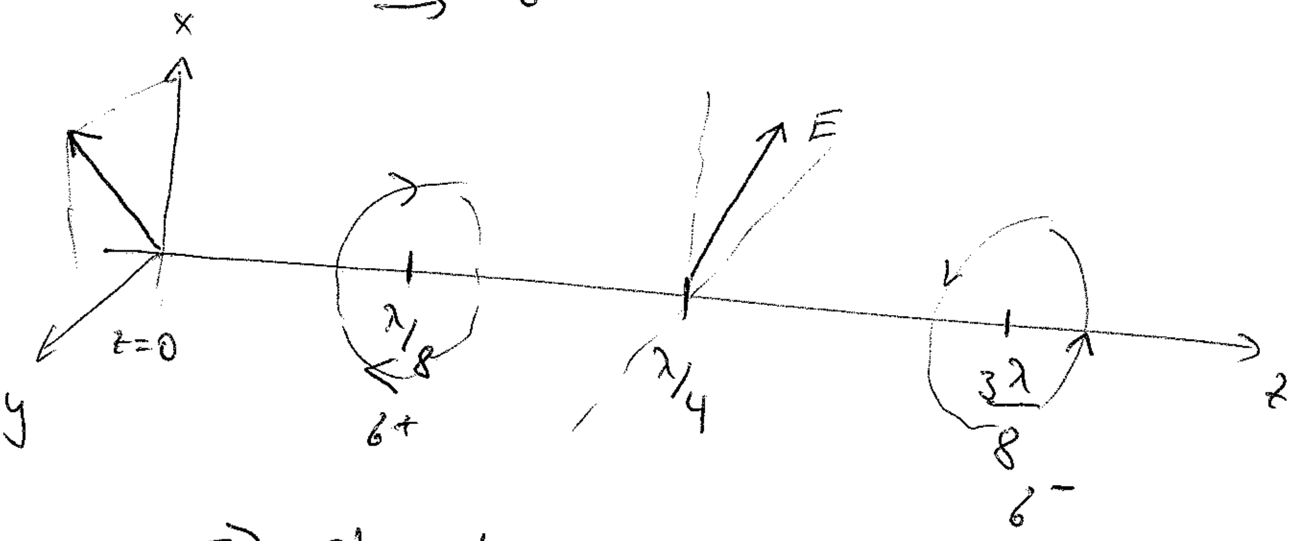
$$E(z = \lambda/8) = E_0 \hat{x} \cos(\omega_L t - \tfrac{\pi}{4}) + E_0 \hat{y} \cos(\omega_L t + \tfrac{\pi}{4})$$

$$= E_0 \hat{x} \sin(\omega_L t + \tfrac{\pi}{4}) + E_0 \hat{y} \cos(\omega_L t + \tfrac{\pi}{4})$$ zirkular pol. Licht σ^+

@ $z = \frac{3\lambda}{8}$

$$E(z = \tfrac{3\lambda}{8}) = E_0 \hat{x} \cos(\omega_L t - \tfrac{3\pi}{4}) + E_0 \hat{y} \cos(\omega_L t + \tfrac{3\pi}{4})$$

$\longrightarrow \sigma^-$



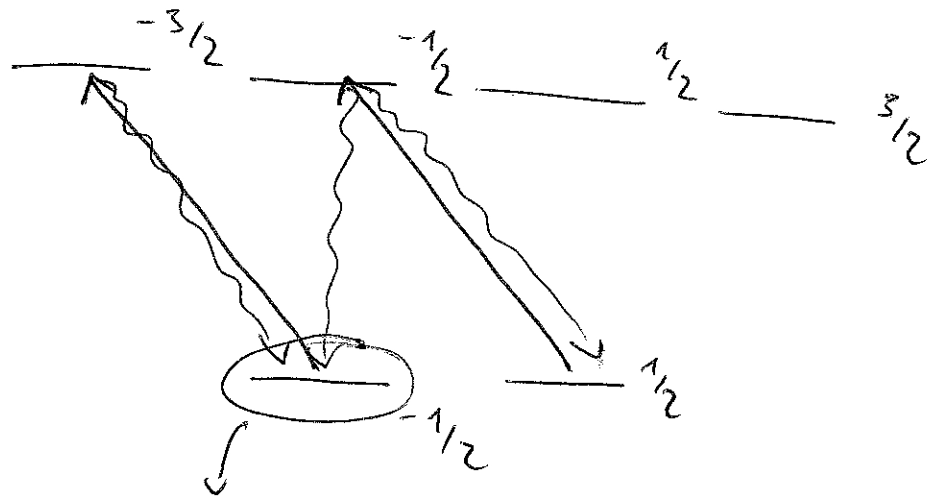
$\Longrightarrow$ standing wave with spatially varying polarization

Consequences

(1) Optical pumping and spatially varying occupation of ground state manifold

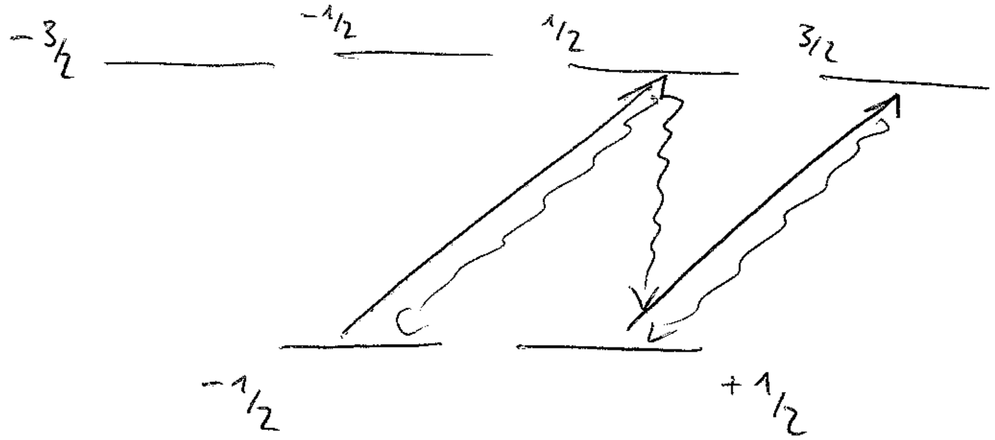
* Atoms have to be slow enough so that they can be pumped into certain sublevels at certain positions while moving through the standing wave

@ σ^-



Population accumulates in ground state $m_g = -1/2$

@ σ^+



Population accumulates in $m_g = +1/2$

(2) light shift

~~What~~

What is the light shift?

Remember last lecture:

Force exerted by the laser on an atom

$$\vec{F}(\vec{R}) = \vec{F}_{RP}(\vec{R}) + \vec{F}_{D}(\vec{R})$$

$$\vec{F}_{RP}(\vec{R}) = -\frac{\hbar s(\vec{R})}{1+s(\vec{R})} \frac{\Gamma}{2} \nabla \Phi(\vec{R})$$

→ radiation pressure force

(~~cooling~~ e.g. used in Doppler cooling)

$$\vec{F}_{D}(\vec{R}) = -\frac{\hbar s(\vec{R})}{1+s(\vec{R})} \delta \frac{\nabla \Omega_1(\vec{R})}{\Omega_1(\vec{R})}$$

Dipole force

$$s(\vec{R}) = \frac{\Omega_1^2/2}{\delta^2 + \Gamma^2/4}$$

~~[illegible]~~

This force can be derived from an associated potential → conservative force

$$\nabla s = 2s \frac{\nabla \Omega_1}{\Omega_1}$$

$$\Longrightarrow \vec{F}_{D} = -\frac{\hbar \delta \cancel{s(\vec{R})}}{1+s(\vec{R})} \frac{\nabla s(\vec{R})}{2\cancel{s(\vec{R})}}$$

$$= -\frac{\hbar \delta}{2} \frac{\nabla s(\vec{R})}{1+s(\vec{R})}$$

$$= -\vec{\nabla}\left[\frac{\hbar \delta}{2} \ln(1+s(\vec{R}))\right]$$

$\vec{F}_D(\vec{r})$ is a conservative force with an associated potential

$\rightarrow$ dipolar potential

$$U_D(\vec{r}) = \frac{\hbar\delta}{2} \ln\left[1 + s(\vec{r})\right]$$

The sign of the potential depends on the sign of δ

× Red detuning ($\delta < 0$)

Atoms will be attracted to regions of higher intensity $\rightarrow$ dipole trap

× Blue detuning ($\delta > 0$)

$\rightarrow$ repulsion from regions of higher intensity

$\rightarrow$ e.g. atom mirror

For $|\delta| \gg \Gamma$ and $s \ll 1$ (weak intensity)

$\Longrightarrow \ln[1+s] \approx s \approx \dfrac{\Omega_1^2}{2\delta^2}$

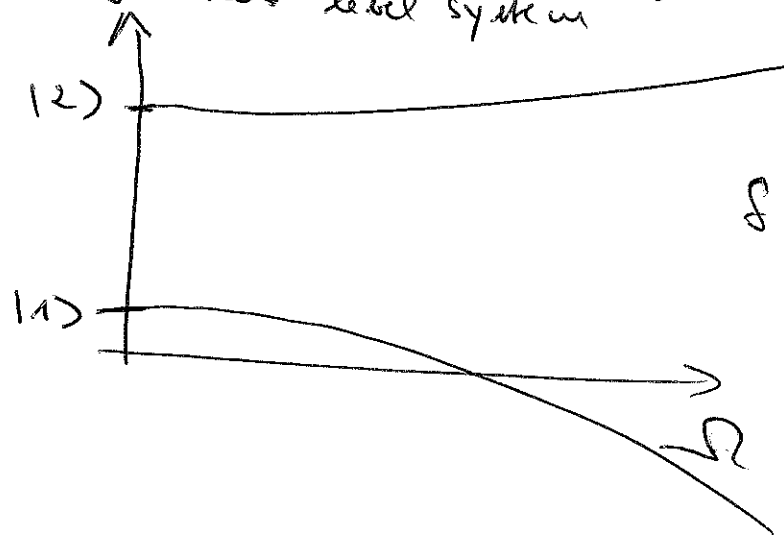
$$\Longrightarrow \boxed{U_D \approx \frac{\hbar\Omega_1^2}{4\delta}}$$

Describe ~~dipole potential~~ ~~in~~ interaction of the light with the

Now, describe the atom-light interaction in a dressed state picture

$\Longrightarrow$

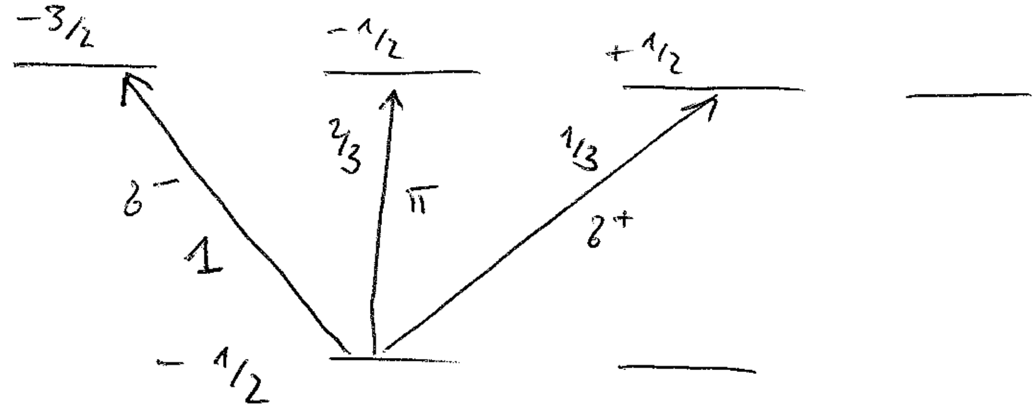
Dipole force, dipole potential leads to a shift of energy eigenvalues e.g. two-level system



Why are we discussing this in the context of subdoppler cooling?

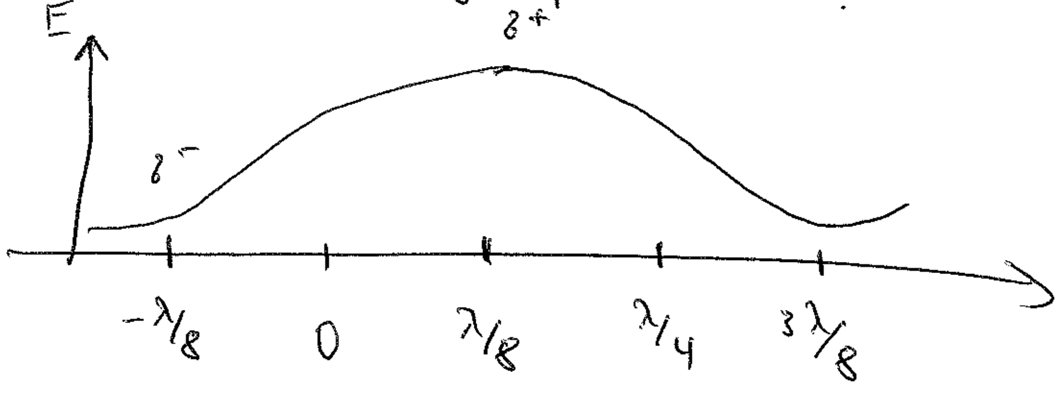
<u>Consider</u>: $m_J = -1/2$ in a standing wave as described above with spatially varying polarization

Relative coupling strengths:



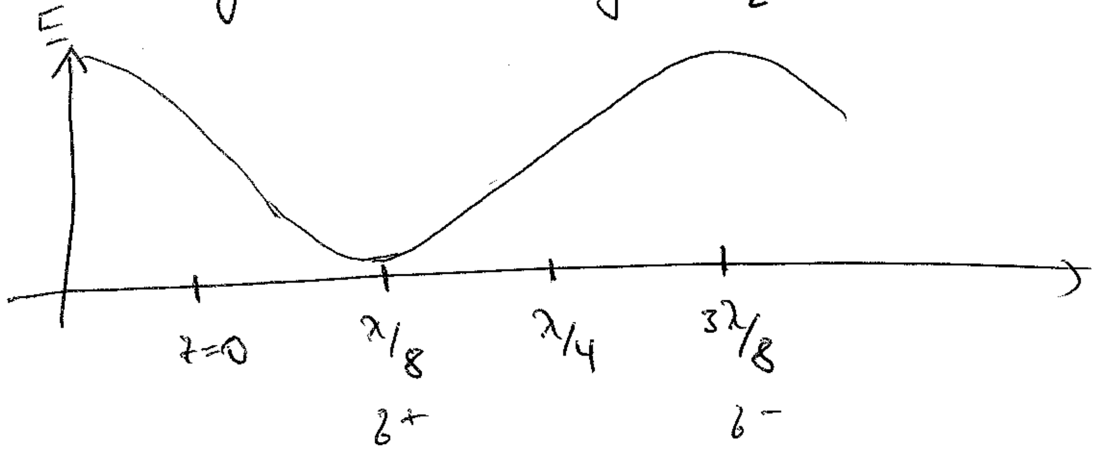
(14)

$\Longrightarrow$ Light shift of $m_g = -1/2$ atom along z-axis, along standing wave with varying polarization:

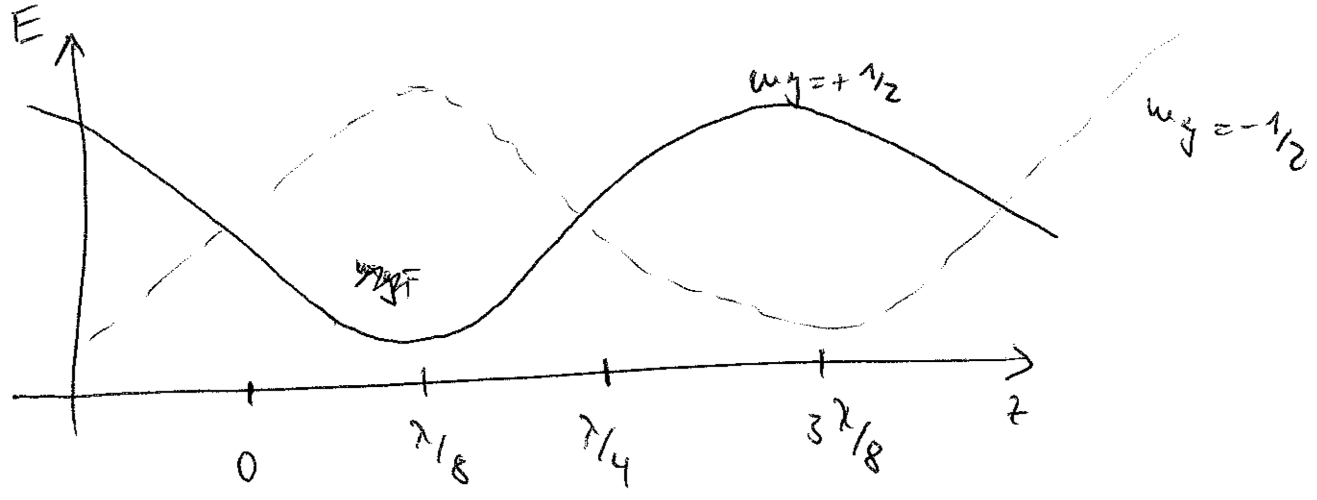


σ^- couples $3\times$ stronger to $m_g = -1/2$ than σ^+

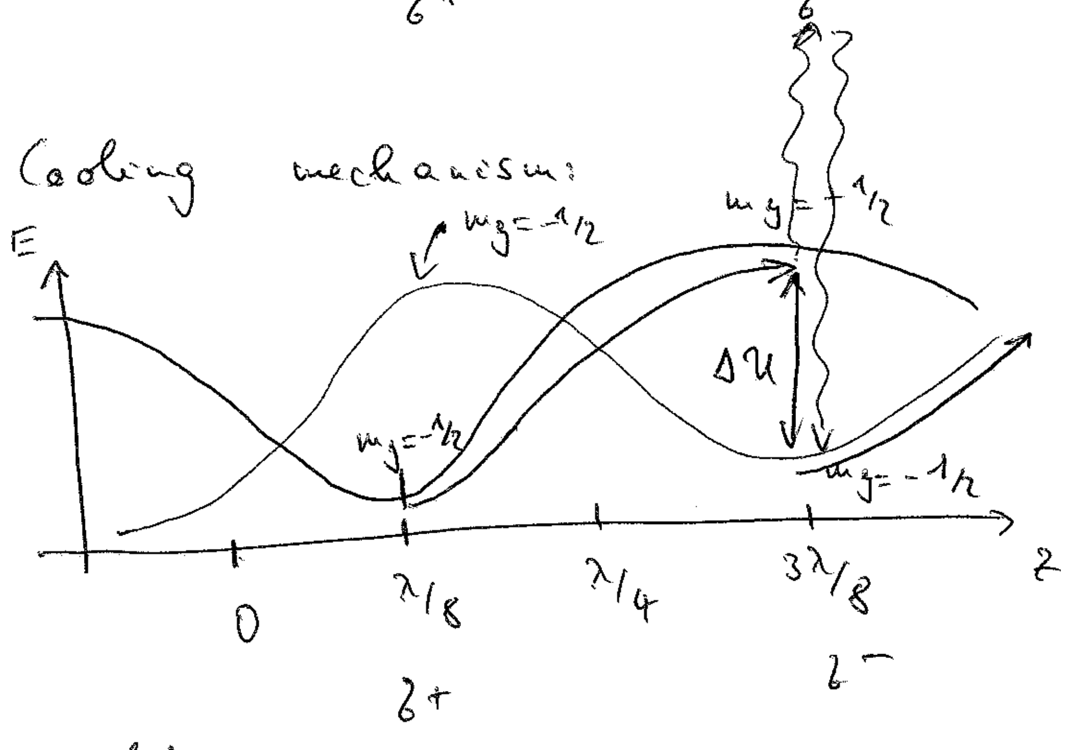
Similarly: $m_g = +1/2$



Summary of light shifts:



Cooling mechanism:



- let us consider an atom in $m_g=+1/2$ @ $\lambda/8$
- $m_g=+1/2$ is low energetically lower than $-1/2$
- atom moving in positive z-direction to $z=3\lambda/8$
- has to climb potential energy hill ΔU
- looses kinetic energy
- at $z=3\lambda/8 \longrightarrow$ $m_g=+1/2$ is being pumped to $m_g=-1/2$
- again climbing up hill

$$\nu_c / \Gamma_{scatt} \sim \frac{1}{k}$$

$$\nu_c \approx \Gamma^3 / 8 \quad \frac{2\Omega_1^2}{\Gamma^2 \delta^2}$$

How large is the force?

$$F = \frac{\Delta E}{\Delta t} \simeq \Delta E\, k \approx \frac{\hbar \Omega^2}{4\delta} k$$

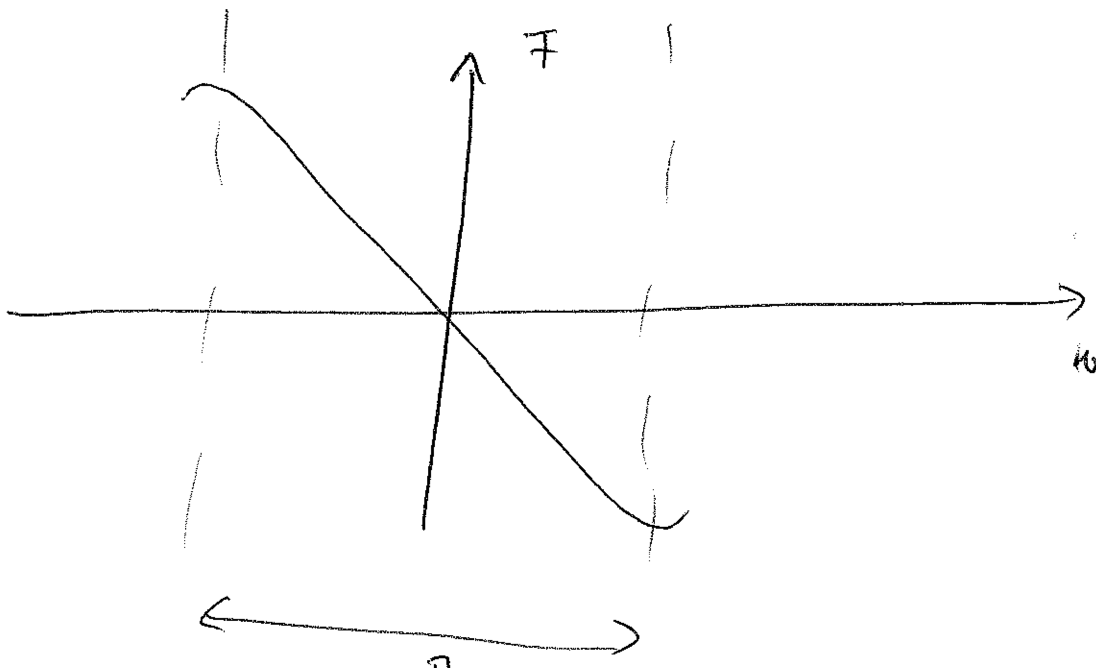
Summarize:

$$F = -\beta\, \nu_c = -\beta \frac{\Gamma^{\cancel{3}}}{\cancel{8}_{\,2}} \frac{\cancel{2}\, \cancel{\Omega_1^2}}{\cancel{\Gamma^2} \cancel{\delta^2}}$$

$$\stackrel{!}{=} \frac{\hbar \cancel{\Omega_1^2}}{\cancel{4}\cancel{\delta}} k$$

$$\Rightarrow -\beta \frac{\Gamma}{\delta} \stackrel{!}{=} \hbar k$$

$$\boxed{\beta = -\hbar k \, \frac{\delta}{\Gamma/k} = -\hbar k^2 \delta/\Gamma}$$

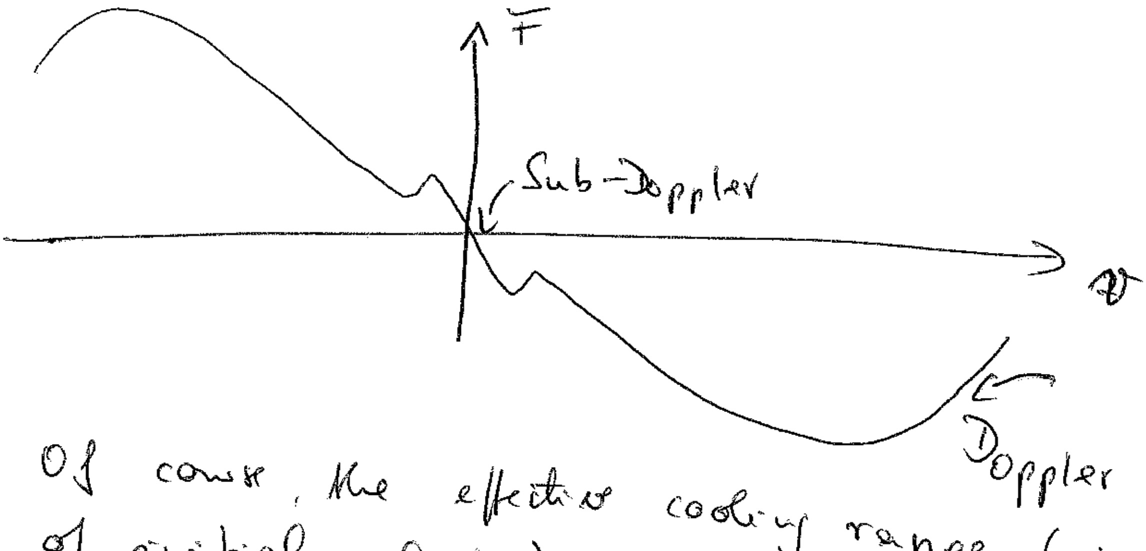


$v \sim \frac{\Gamma_{scatt}}{k}$

not Γ/k as for Doppler cooling

Some additional notes

× The maximum damping coefficient is a factor of $2\delta/\Gamma$ larger than for Doppler cooling ($\sim \hbar k^2$)



× Of course, the effective cooling range (in terms of initial velocity) is limited

Cooling limit for Sub-Doppler cooling

× The system will be in equilibrium when $\Delta U \sim E_r$

$$\Rightarrow T \sim \frac{E_r}{k_B} \qquad E_r = \frac{\hbar^2 k^2}{2M}$$

$$Na \sim E_r \sim \underline{\underline{2.4\,\mu K}}$$