1.6.2. $\sigma^+\sigma^-$ Polarization gradient scheme

× two counter propagating beams with σ^+ and σ^- polarization, respectively

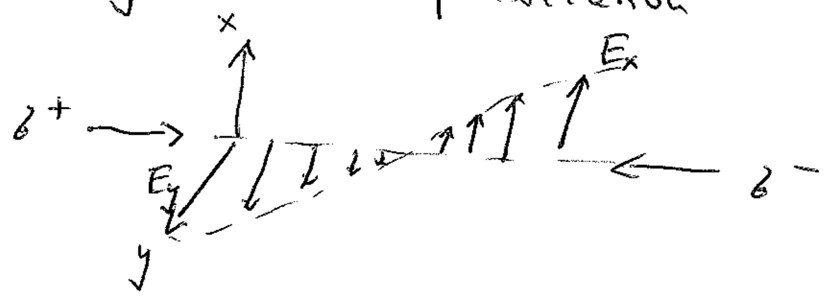
both having positive helicity

σ_R with $\vec{k} \parallel \vec{e}_z \equiv \sigma^+$

σ_R with $\vec{k} \parallel -\vec{e}_z \equiv \sigma^-$

What happens with the polarization of the standing wave?

Rotating linear polarization



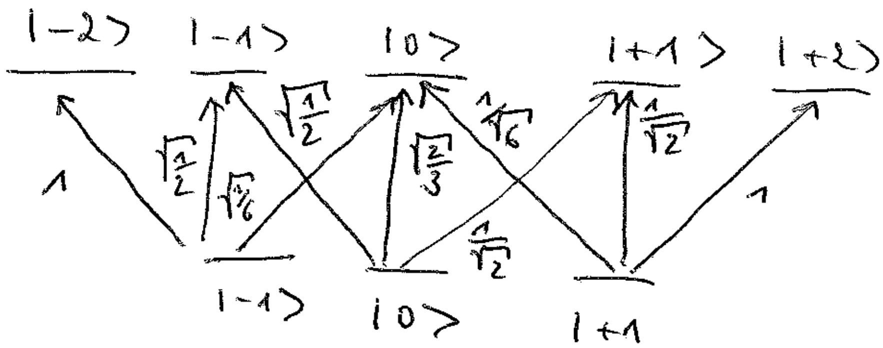
$$\vec{E}(\vec{r},t) = E_0 \sin(kz)\, e^{i\omega_L t}\, \vec{e}_x + E_0 \cos(kz)\, e^{i\omega_L t}\, \vec{e}_y + c.c.$$

Standing wave, linearly polarized with a polarization rotating around Oz. $\longrightarrow$ polarization gradient in space.

Intuitive picture for resulting damping force

(1) Consider atom at rest

Quantization axis along E_y



Population in different substates

Rate equations

$$\dot{\rho}_{00}\tau_p = -\frac{2}{3}\cdot\frac{1}{6}\rho_{00} - \frac{2}{3}\cdot\frac{1}{6}\rho_{00} + \frac{1}{2}\cdot\frac{1}{2}\rho_{11} + \frac{1}{2}\cdot\frac{1}{2}\rho_{-1-1}$$

similar equations for ρ_{+1+1} ρ_{-1-1}

Because of ~~comp~~ symmetry

$\rho_{11} = \rho_{-1-1}$

$$\Rightarrow \dot{\rho}_{00}\tau_p = -\frac{2}{3}\cdot\frac{1}{6}\rho_{00}\cdot 2 + \frac{1}{2}\cdot\frac{1}{2}\rho_{11}\cdot 2$$

and $\rho_{00} + 2\rho_{11} = 1$

Now ask for $\dot{\rho}_{00} = 0$

$\Rightarrow \rho_{00} = \frac{9}{17} \qquad \rho_{11} = \rho_{-1-1} = \frac{4}{17}$

(2) Now, consider an atom moving with v along z

$\Longrightarrow$ The atom encounters a rotating polarization

$$\mathcal{E}(t) = \begin{pmatrix} \sin kvt \\ \cos kvt \end{pmatrix}$$

Does this give rise to a friction force?

Qualitative picture.....

Transform to a slowly rotating frame in which the polarization vector is constant.

Unitary transformation

$T(t) = e^{\frac{i}{\hbar} kvt J_z}$ (rotation by angle $\varphi = -kvt$

Transform now the interaction part of the Hamiltonian in the lab frame

$$H^{AL} = -\vec{d} \cdot \vec{E} = -d_x E_0 \sin kvt - d_y E_0 \cos kvt$$

into the new frame

$$T^{+}(t) H_{ED} T(t) = \ldots\ldots = -d_y E_0$$

The transformed interaction takes place between the atomic dipole moment and a laser field with a fixed polarization along $\vec{e_y}$

$\Longrightarrow$ Transformed Hamiltonian

$$H' = T^\dagger H T - i\hbar T^\dagger \frac{dT}{dt}$$

$$= -d_y E + kv J_z$$

fictitious magnetic field $B_z^{eff} = kv\hbar/\mu_B$ pointing along z-axis $\perp$ quantization axis

$\Longrightarrow$ coupling between $\overline{|+1\rangle}$, $\overline{|-1\rangle}$, $\overline{|0\rangle}$

$\Longrightarrow$ steady state population difference between the eigenstates $|\pm 1\rangle$ of J_z.

$$\rho_{11} - \rho_{-1-1} = \frac{40}{17} \frac{kv}{\Delta E_-} \qquad \Delta E_- = \frac{\hbar |\Omega|^2}{\delta}$$

Cooling mechanism:

Suppose an atom is moving along the positive z axis, $v > 0$, $\delta < 0$

(1) $|-1\rangle_z$ more populated than $|+1\rangle_z$

$$\rho_{-1-1} > \rho_{+1+1}$$

(2) $|-1\rangle_z$ scatters σ^- 6x more effectively than σ^+ light.

} unbalanced rad. pressure

* Effective cooling since the faster the atom moves, the higher is the population difference and hence the probability to absorb a counter-propagating photon.

$$\vec{F} = \hbar \vec{k} \left(\rho_{11} \Gamma_{scatt} - \rho_{-1-1} \Gamma_{scatt} \right)$$

$$\propto \frac{\hbar k^2 \Gamma_{scatt}}{\Delta E_-} \vec{v}$$

* for $\delta < 0 \Rightarrow \Delta E_- < 0$

  $\Rightarrow$ force opposes motion

* friction coefficient $\sim \dfrac{\Gamma_{scatt}}{\Delta E_-} \sim \dfrac{1}{\delta}$

  $\Rightarrow$ much smaller for large detunings than the friction coefficient found in lin $\perp$ lin case

  $\left(\sim \dfrac{\Delta E_-}{\Gamma_{scatt}} \sim \delta \right)$

* diffusion coefficient also smaller

  $\Rightarrow$ same equilibrium temperature $\sim E_r$

Cooling into a Moving Frame and Atomic Fountain

× Apply an additional magnetic field in the z-direction.

As a consequence, levels are Zeeman shifted
$\Delta E = \frac{\mu_B B_z}{\hbar}(g_e m_e - g_g m_g)$

× Difference in steady state ground state populations.

× Choose $\vec{B}_z = B_0(0,0,-1) \Rightarrow \rho_{11} > \rho_{-1-1}$
$\vec{B}_z = B_0(0,0,1) \Rightarrow \rho_{-1-1} < \rho_{11}$

× Real magnetic field induces population difference for all velocities, also $v=0$

Now choose $B = B_0 \; (0,0,-1)$ such that it opposes the motion induced population difference for $v > 0$

Then f.ex. for $v=0$ $\quad \rho_{11} > \rho_{-1-1}$

$\Rightarrow$ Atoms are being accelerated to v_z due to a stronger absorption of σ^+

$\Rightarrow$ ρ_{11} will decrease until forces due to motion and magnetic field cancel

$$\Rightarrow \frac{k v \hbar}{\mu_B} = B \Rightarrow v = \frac{\mu_B B}{\hbar k} \Rightarrow v \approx 1.4 \; @ B = 1G$$

Atoms will be "cooled" into a moving frame.

Way towards quantum degeneracy in atomic gases

× atomic vapor at high temperature 300 K – 1000 K

× magneto-optical trapping

accumulating atoms ↙ ↘ cooling ~ T_D

× Sub-Doppler cooling, molasses ~ $E_r \sim kT$

Missing step: cooling without light forces

× magnetic trapping

× evaporative cooling

2. Trapping and evaporative cooling of neutral atoms

2.1. Magnetic trapping of neutral atoms

× MOT is an example of a dissipative trap. In addition to the trapping force, we have friction. At low temperature, spontaneous emission induces heating

× Magnetic trap: conservative trap based on the Zeeman effect

Interaction of the magnetic moment $\vec{\mu}$ of an atom with an external magnetic field

$$V = -\vec{\mu}\cdot\vec{B} \qquad \vec{\mu}_F = -g_F\,\mu_B\,\frac{\vec{F}}{\hbar}$$

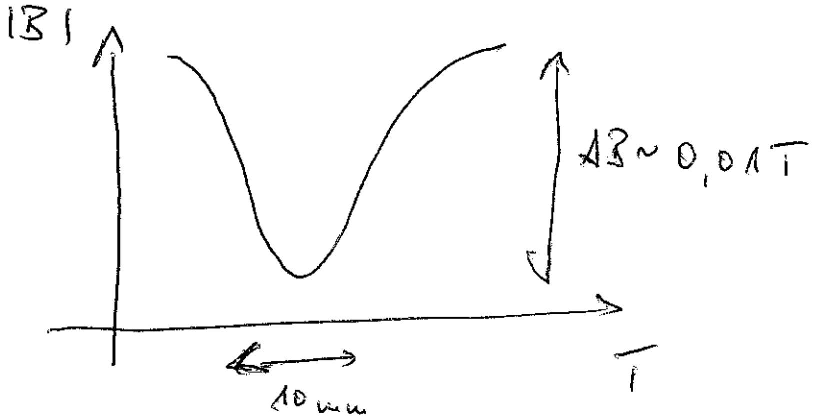
$$V = -\vec{\mu}\cdot\vec{B} = g_F\,\mu_B\,\frac{\vec{F}}{\hbar}\cdot\vec{B} = g_F\,\mu_B\,m_F\,B$$

Force

$$F = -g_F\,\mu_B\,m_F\,\nabla B$$

Demonstration of this force: Stern-Gerlach-Experiment

<u>Estimate depth of magnetic trap</u>



$$k_B T \sim \mu_B B$$

$$T \sim 0.67\,\frac{K}{T}\cdot 0.01\,T$$

$$= 0.0067\,K$$

$$\approx 10\,mK$$

Doppler cooled atoms can be trapped

Required magnetic field gradient

$$\sim \frac{0{,}01\,T}{10\,mm} \sim \frac{1\,T}{m}$$ is possible!

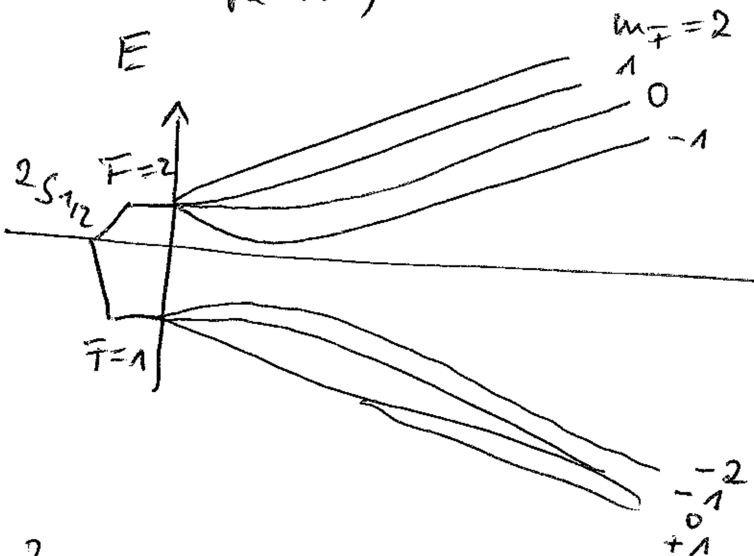
Realization of a magnetic trap

* for storing atoms, you need an extremum in the magnetic field (minimum or maximum)
* Maxwell equations:
  for quasistatic fields, you can only prepare magnetic field minima
* magnetic trap on the basis of static fields has to work with a magnetic field minimum

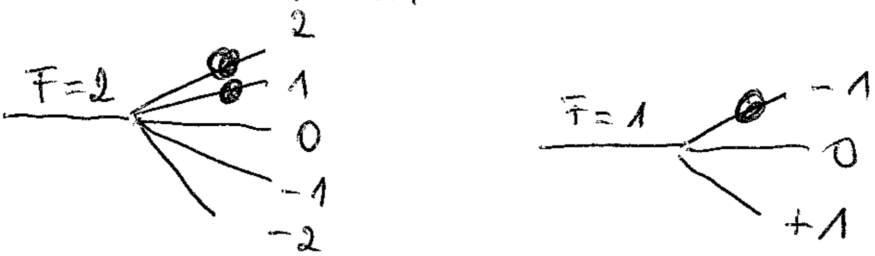
$\Longrightarrow$ only low-field seeking states can be trapped

$g_F \, m_F > 0$

(the smaller the magnetic field, the smaller the energy of the atom in the field)



Zeeman regime:



× magnetic field direction is not relevant, only the absolute value

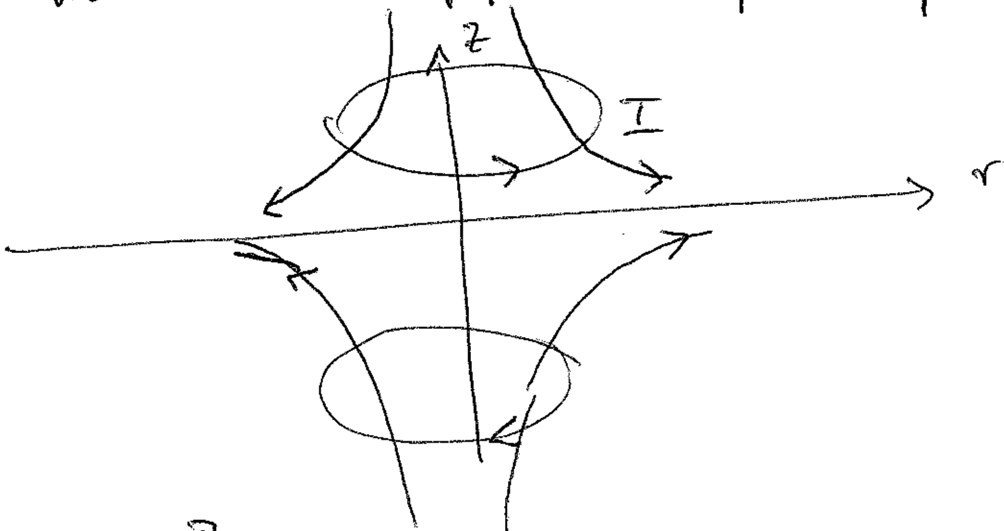
→ magnetic moment follows adiabatically the magnetic field direction

<u>But</u>: this is only possible if

Larmor frequency ≫ typical trap oscillation ~~frequency~~

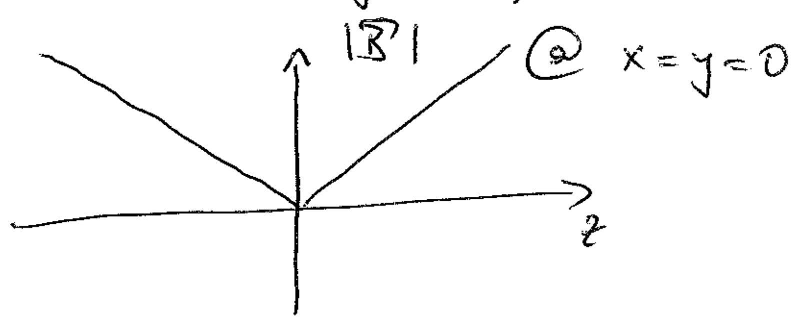
<u>Simplest magnetic trap</u>

Anti-Helmholtz configuration (quadrupole trap)



$$B(x,y,z) = B'\,(x\vec{e}_x + y\,\vec{e}_y - 2z\,\vec{e}_z)$$

$$|\vec{B}| = B'\,(x^2+y^2+4z^2)^{1/2}$$



Issue

$B = 0$ @ $z = r = 0$

$\Rightarrow$ Spin flips, Majorana losses

Why?

Stable classical trajectories where the spin can adiabatically follow the magnetic field only for

$\omega_L \gg \omega_T$

In $z = 0$ plane, we get a stable classical trajectory if

$\mu \nabla B = \frac{M v^2}{\rho}$ $\qquad \rho = \sqrt{x^2 + y^2}$

force exerted by magnetic field on atoms

centripetal force

$\Rightarrow v = \sqrt{\frac{\mu \nabla B \rho}{M}} = \sqrt{\rho \cdot a}$ $\qquad a = \frac{\mu \nabla B}{M}$

centripetal acceleration by magnetic field gradient

No spin flips, if the spins are following adiabatically the magnetic field

$$\omega_L = \frac{\mu_B B}{\hbar} \gg \omega_T = \frac{v}{\rho} = \frac{\sqrt{\rho \cdot a}}{\rho} = \sqrt{\frac{a}{\rho}}$$

Larmor frequency

angular velocity of the atoms

$$\omega_L = \frac{\mu_B \nabla B \cdot \rho}{\hbar} \gg \omega_T = \sqrt{\frac{a}{\rho}}$$

$$\frac{a \cdot \rho M}{\hbar} \gg \sqrt{\frac{a}{\rho}}$$

$$\Rightarrow \rho^3 \gg \frac{\hbar^2}{M^2 a} \quad \text{or} \quad \rho \gg \sqrt[3]{\frac{\hbar^2}{M^2 a}}$$

What does this mean for the velocity

$$v^2 = \rho a \gg \sqrt[3]{\frac{\hbar^2}{M^2}\frac{1}{a}} \cdot a$$

$$= \sqrt[3]{\frac{\hbar^2}{M^2}} \cdot a^{2/3}$$

$$v \gg \sqrt[3]{\frac{\hbar}{M} a}$$

Examples: $\nabla B = 100 \frac{G}{cm}$

$10^4 G = 1 T$

$\Rightarrow a \approx 250 \frac{m}{s^2}$ for Na

$\Rightarrow v \gg 1 \frac{cm}{s}$ and $\rho \gg 1 \mu m$

$\downarrow$

$T \sim 120 nK$

If the atomic velocity goes beyond $1 \frac{cm}{s} \rightarrow$ losses

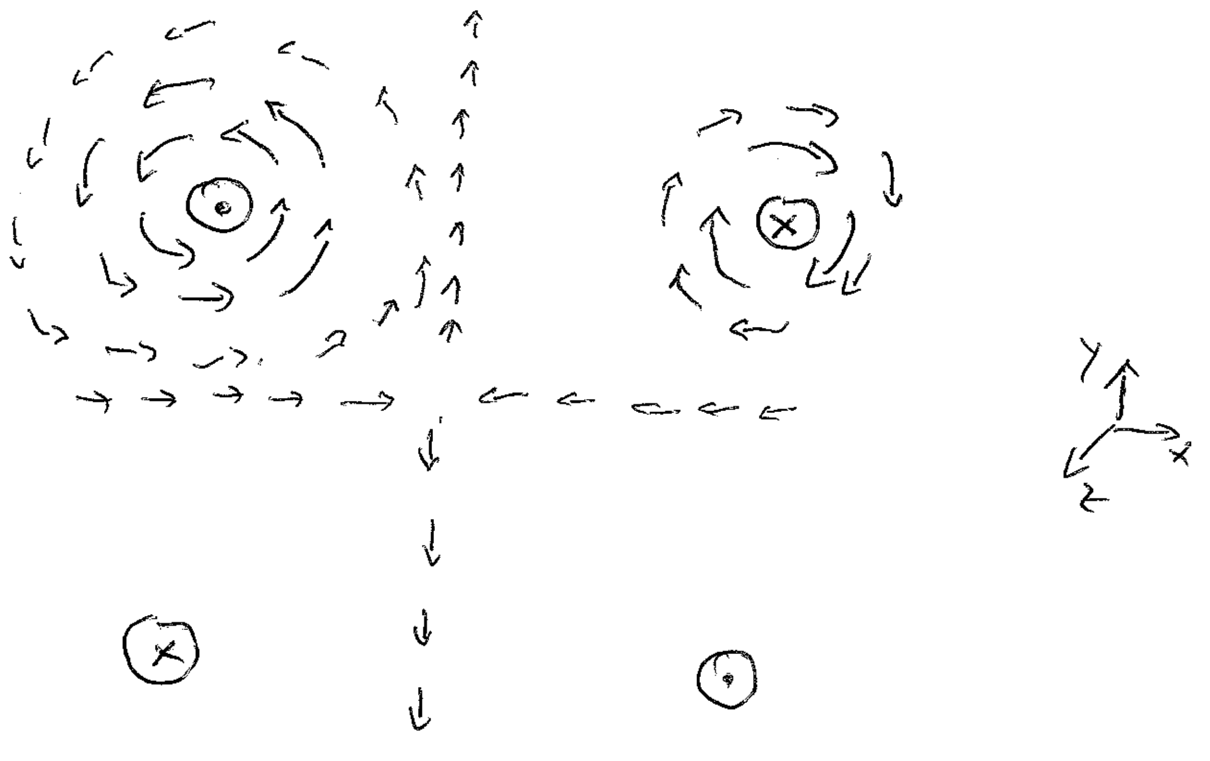
How to avoid Majorana losses?

(1) Close the ~~hole~~ hole at $z = \rho = 0$ by a blue detuned dipole trap pushing the atoms out of the critical volume

(2) Ioffe Pritchard trap or other traps without a magnetic field 0 in the center

The Ioffe Pritchard trap

(1) Four rods / wires with currents as sketched below



× neighbouring wires opposite current direction

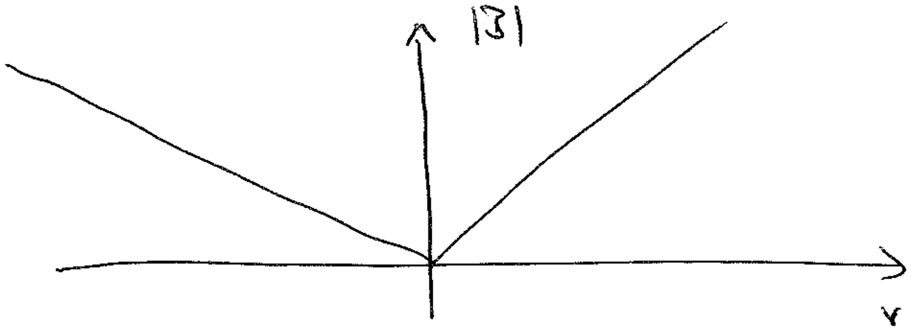
× no field gradient in z-direction

$$\frac{dB_z}{dz} = 0$$

× $\operatorname{div} \vec{B} = 0 \Rightarrow \frac{dB_x}{dx} = -\frac{dB_y}{dy} = b'$

$$\Rightarrow \quad \vec{B} = b' \left(x \vec{e}_x - y \vec{e}_y \right)$$

$$|\vec{B}| = b' \left(x^2 + y^2 \right)^{1/2} = b' r$$



× Force

$$\vec{F} = - \nabla V = - g_F \mu_B m_F b' \hat{e}_r$$

× force in the radial direction

× trap in x-y plane for $g_F m_F > 0$

<u>but</u> $B = 0$ at $x = y = 0$

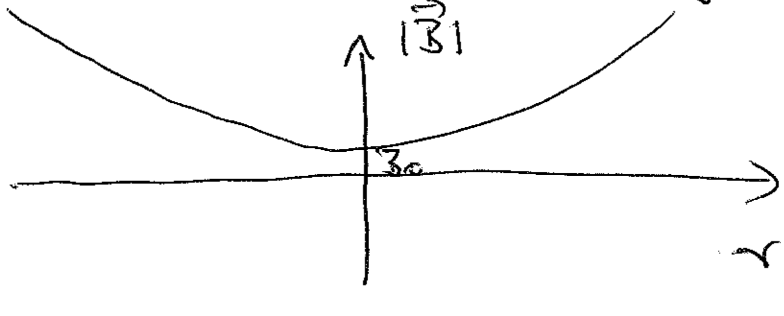
$\Rightarrow$ Majorana losses

(2) Add homogeneous field along z-axis for example by adding Helmholtz coils

$$\vec{B}_0 = B_0 \vec{e}_z$$

$$\Rightarrow |\vec{B}| = \left\{ B_0^2 + (b' r)^2 \right\}^{1/2} \simeq B_0 + \frac{b' r^2}{2 B_0}$$

for $b' r \ll B_0$



$$V(r) = V_0 + \frac{1}{2} m \omega_r^2 r^2 = g_F m_F \mu_B \left(B_0 + \frac{b'^2 r^2}{2B_0}\right)$$

$$\omega_r^2 = \sqrt{\frac{g_F m_F \mu_B}{m B_0}} \; b'$$

× the ~~large~~ smaller B_0 the large the trap frequency

(3) Confinement in z-direction

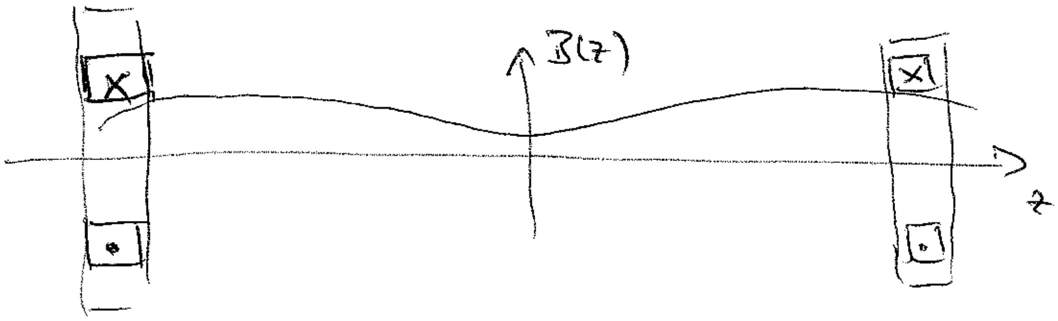
× Add pinch coils

× two coils

× current flow in the same direction

× but distance between coils much large than Helmholtz

⟹ B-field has a minimum



$$B_{pinch}(z) = B_{pinch}(0) + \frac{d^2 B_z}{dz^2} \frac{z^2}{2}$$

field curvature

→ confinement in axial (z-direction)

Some estimates

$b' \approx 3\,\frac{T}{m}$ $\qquad B_0 = 3\times 10^{-4}\,T$

$b'' = \frac{d^2 B}{dz^2} \simeq 300\,\frac{T}{m^2}$

$\Longrightarrow \quad \omega_z/2\pi = 15\,Hz$ for Na

$\omega_r/2\pi = 250\,Hz$