two counter propagating beams with
$\mathbf{a}^+$ and $\mathbf{a}^-$ polarization, respectively

$$\mathbf{a}^+ \rightarrow \mathbf{a}^-$$

both having positive helicity
$\mathbf{b}_x$ with $\mathbf{b}_x \parallel \mathbf{a}^+ = \mathbf{a}^+$
$\mathbf{b}_z$ with $\mathbf{b}_z \parallel -\mathbf{a}^+ = \mathbf{a}^-$

What happens with the polarization of the standing wave?

Rotating linear polarization

$$\mathbf{b}^+ \rightarrow \mathbf{b}^-$$

$$\mathbf{E}(s,t) = E_0 \sin(kz) e^{i\omega t} \mathbf{a}_x + E_0 \cos(kz) e^{i\omega t} \mathbf{a}_y + \text{c.c.}$$

Standing wave, linearly polarized
with a polarization rotating around $\mathbf{a}_z \rightarrow$ polarization gradual
in space.
(1) Consider a planar model

Quantization axes along $E_y$

Population in different substates

Rate equations

$$\dot{S}_{60} \approx \frac{2}{3} \cdot \frac{1}{6} S_{60} - \frac{2}{3} \cdot \frac{1}{6} S_{60} + \frac{1}{2} \cdot \frac{1}{2} S_{11} + \frac{1}{2} - \frac{1}{2} S_{-1-1}$$

Similar equations for $S_{-1-1}$, $S_{11}$

Because of symmetry

$$S_{11} = S_{-1-1}$$

$$\Rightarrow \dot{S}_{60} \approx \frac{2}{3} \cdot \frac{1}{6} S_{60} + \frac{1}{2} \cdot \frac{1}{2} S_{11} + 2$$

and $S_{60} + 2 S_{11} = 1$

Now ask for $\dot{S}_{60} = 0$
\[ S_0 = \frac{g}{4\pi} \quad S_n = S_{n-1} - \frac{1}{4\pi} \]

(2) Now, consider an atom moving with velocity \( v \) along \( z \).

\[ \Rightarrow \text{The atom encounters a rotating polarization} \]

\[ \mathbf{E}(t) = \begin{pmatrix} \sin k_0 t \\ \cos k_0 t \end{pmatrix} \]

Does this give rise to a friction force?

Qualitative physics.

Transform to a slowly rotating frame in which the polarization vector is constant.

\[ \text{Unitary transformation} \]

\[ T(t) = e^{\frac{i}{\hbar} k_0 t} \quad (\text{rotation by angle } \varphi = -k_0 t) \]

Transform now the interaction part of the Hamiltonian in the lab frame

\[ H^{\text{int}} = -\mathbf{d} \cdot \mathbf{E} = -d_x E_0 \sin k_0 t - d_y E_0 \cos k_0 t \]

into the new frame

\[ \mathbf{T}^+(t) H_{\text{int}} \mathbf{T}(t) = \ldots = -d_y E_0 \]
The transformed interaction takes place between the atomic dipole moment and a laser field with a fixed polarization along $\vec{E}_y$.

$$\Rightarrow\text{Transformed Hamiltonian}$$

$$H' = \hat{T} + H - ic \hat{T} \frac{d\hat{1}}{dt}$$

$$= -\alpha y \hat{E} + \hbar \omega \hat{J}_z$$

A fictitious magnetic field $\hat{B}' = \hbar \omega / \alpha$ points along the $z$-axis, perpendicular to the quantization axes.

$$\Rightarrow\text{Coupling between } |y\uparrow\uparrow\rangle, |y\uparrow\downarrow\rangle, |10\rangle$$

$$\Rightarrow\text{Steady state population difference between the eigenstates } |1\pm1\rangle \text{ of } \hat{y}_2.$$

$$\Delta_{-} = \Delta_{-} - \Delta_{-1} = \frac{\hbar \omega}{\Delta E} \frac{E}{E} \Delta E = \frac{\hbar \omega |\Delta E|^2}{\Delta E}$$
Cooling mechanism:
Suppose an atom is moving along the positive \( z \) axis, \( \theta > 0 \) & \( \phi < 0 \)

\[
\begin{align*}
\text{(1)} \quad |\uparrow \rangle_z & \quad \text{more populated than} \quad |\downarrow \rangle_z \\
S_{-1-1} & > S_{+1+1} \\
\text{(2)} \quad |\uparrow \rangle_z \quad \text{scatters 8 - 6} x \quad \text{more effectively than} \quad |\downarrow \rangle_z \text{light.}
\end{align*}
\]

Effective cooling: Since the facts the atom moves, the lighter is the population difference and hence the probability to absorb a counter-propagating photon.

\[
F = \frac{k^2}{\hbar^2} \left( S_{11} \Gamma_{\text{scatt}} - S_{-1-1} \Gamma_{\text{scatt}} \right)
\]

\[
< \frac{k^2 \Gamma_{\text{scatt}}}{\Delta E_r} >
\]
* for \( s < 0 \) \( \Rightarrow \Delta E < 0 \)
  \( \Rightarrow \) force opposes motion

* friction coefficient \( n \) \( \frac{n_{\text{scatter}}}{\Delta E} \sim \frac{n}{\Delta E} \)
  \( \Rightarrow \) much smaller for large-decayings than the friction coefficient found
  in Lin-Lin case

* diffusion coefficient also smaller
  \( \Rightarrow \) same equilibrium temperature \( n \) \( E \)
Apply an additional magnetic field in the \(z\)-direction.

As a consequence, levels are Zeeman shifted:

\[ A = \frac{\mu_3 B_2}{h} (g_{\text{we}} - g_{\text{mg}}) \]

Difference in steady state ground state populations.

\[ \frac{3}{2} = \frac{3}{0} (0, 0, -1) \Rightarrow S_{m} > S_{-1-1} \]
\[ \frac{1}{2} = \frac{3}{0} (0, 0, 1) \Rightarrow S_{-1-1} < S_{m} \]

Choose \( B = B_0 (0, 0, -1) \) and let it oppose the photon induced population difference for all \( m_{2} \) values.

Then \( \Delta x \) for \( m_{2} = 0 \):
\[ S_{m} > S_{-1-1} \]

\[ \Rightarrow \text{Mons are being accelerated to 0} \]

\[ \Rightarrow \text{Mons will decrease until forces due to} \] 
\[ \text{wether and magnetic field cancel} \]

\[ \Rightarrow \frac{\mu_0 B}{\mu B} = \beta \Rightarrow \beta = \frac{\mu_3 B}{h A} \Rightarrow A \approx 1.4 \]

After closing
Atoms will be "cooled" into a moving frame.
Way towards quantum degeneracy in atomic gates

- atomic vapor at high temperature 300K - 1000K
-wave optical trapping
  accumulating → cooling
- sub-Doppler cooling, molasses \( \propto E_c \propto kT \)

missing step: cooling without light forces

- wave optical trapping
- evaporative cooling

2. Trapping and evaporative cooling of neutral atoms

2.1. Maseric trapping of neutral atoms

- Maser is an example of a dispersive trap. In addition to the trapping force, we have friction. At low temperatures, spontaneous emission induces heating

- maser trap: capacitive trap based on the Zeeman effect
Interaction of the magnetic moment $\mu$ of an atom with an external magnetic field

$$V = -\mu \cdot B = g_\pi \mu_B \frac{F}{\hbar} = g_\pi \mu_B B$$

Force

$$F = -g_\pi \mu_B B$$

Demonstration of this force: Sisyphus - Experiments

Estimate depth of magnetic trap

$$V \sim 0.01 T$$

$$T \approx 0.67 \frac{K}{0.01 T} = 0.067 K$$

Joffler cooled atoms can be trapped

Required magnetic field for detail

$$\sim \frac{0.01 T}{10 \mu m} \sim \frac{1 T}{1 m}$$

is possible!
realization of a magnetic trap

- for storing atoms, you need an extremum in the magnetic field (minimum or maximum)
- Maxwell equations:
  - for quasi-static fields, you can only prepare magnetic field minima
  - magnetic trap on the basis of static fields has to work with a magnetic field maximum

$\Rightarrow$ only low-field seeking states can be trapped
$g_{\text{fs}} m_f > 0$

( the smaller the magnetic field, the smaller the energy of the atom in the field )

\[ E \]

\[ F = 2 \]
\[ F = 1 \]
\[ \tilde{F} = 1 \]
\[ \tilde{F} = 0 \]
\[ \tilde{F} = -1 \]
\[ \tilde{F} = -2 \]

\[ m_f = 2 \]
\[ m_f = 1 \]
\[ m_f = 0 \]
\[ m_f = -1 \]
\[ m_f = -2 \]

Zeeman effect:

\[ F = 2 \]
\[ F = 1 \]
\[ F = 0 \]
\[ F = -1 \]
\[ F = -2 \]
x magnetic field direction is not relevant, only the absolute value

→ magnetic moment follows adiabatically the magnetic field direction

But: this is only possible if

\[ \text{Larmor frequency} \gg \text{typical trap oscillation frequency} \]

**Simplified magnetic trap**

\[ \text{A} - \text{Helmholtz configuration (quadrupole trap)} \]

\[ B(x, y, z) = \frac{1}{3} (x \hat{e}_x + y \hat{e}_y - 2z \hat{e}_z) \]

\[ |B| = \frac{1}{3} (x^2 + y^2 + 4z^2)^{1/2} \]

@ \[ x = y = 0 \]
Issue

$B = 0 \Rightarrow t = x = 0$

$\Rightarrow$ Spin flips, Majorana losses

Why?

Stable classical trajectories where the spin can adiabatically follow the magnetic field only for

$\omega_L \Rightarrow \omega_1$

In $z = 0$ plane, we get a stable classical trajectory if

$\mu B_0 = \frac{\hbar \omega_1}{8e}$

$S = \sqrt{x^2 + y^2}$

force exerted by magnetic field on atom

$\Rightarrow \nu = \sqrt{\frac{\mu B_0 S}{M}} = \sqrt{S, \nu}$

$\alpha = \frac{\mu B_0}{M}$

Centripetal acceleration by magnetic field gradient

No spin flips, if the spins are following adiabatically the magnetic field.
\[ \omega_L = \frac{\mu B^2}{\hbar} \Rightarrow \omega_c = \frac{e}{\hbar} = \frac{\sqrt{\frac{\hbar}{2m}}} = \sqrt{\frac{a}{\hbar}} \]

Larmor frequency

\[ \omega_L = \frac{\sqrt{\frac{4\pi\varepsilon_0}{\mu}}}{\hbar} \Rightarrow \omega_c = \sqrt{\frac{a}{\hbar}} \]

\[ \frac{a \cdot s \cdot m}{\hbar} \Rightarrow \sqrt{\frac{a}{\hbar}} \]

\[ \Rightarrow s^3 \gg \frac{\hbar^2 a}{\hbar^3 a} \quad \text{or} \quad s \ll \sqrt[3]{\frac{\hbar^2 a}{\hbar^3 a}} \]

What does this mean for the velocity?

\[ s^2 = sa \Rightarrow \frac{3^{\frac{1}{2}}}{\hbar} \frac{a}{\hbar} \cdot a \]

\[ = 3^{\frac{1}{3}} \frac{\hbar^2}{\hbar^2} \cdot a^{3/3} \]

\[ \Rightarrow \sqrt{\frac{\hbar}{m}} \cdot a \]

\[ \text{Examples:} \quad 18 = 100 \frac{\text{cm}}{\text{cm}} \]

\[ 10^4 \text{ cm} = 1 \text{ cm} \]

\[ \Rightarrow a \approx 250 \frac{\text{m}}{\text{s}^2} \text{ for Na} \]

\[ \Rightarrow a \gg 1 \frac{\text{cm}}{\text{s}} \quad \text{and} \quad s \gg 1 \text{ cm} \]

\[ \Rightarrow T \approx 120 \text{ nK} \]

If the atomic velocity goes beyond \( 1 \frac{\text{cm}}{\text{s}} \rightarrow \text{cooler} \)
How to avoid Majorana losses?

1. Close the whole at \( z = \theta = 0 \) by a blue defined depolar trap pushing the atoms out of the critical volume.

2. Jaffe Pritchard trap or other traps without a magnetic field \( 0 \) in the center.

The Jaffe Pritchard trap

1. Four rods/wires with currents as the shaded below.

\[ \frac{d B_x}{dt} = 0 \]

\[ d(x \times B) = 0 \Rightarrow \frac{d B_x}{dx} = -\frac{d B_y}{dy} = b \]
\[ \vec{B} = b' \left( x \vec{e}_x - y \vec{e}_y \right) \]
\[ |\vec{B}| = b' \left( x^2 + y^2 \right)^{1/2} = b' \sqrt{r} \]

\[
\vec{F} = -4\pi V = -\frac{g_{\tau}}{\mu_3} \frac{w}{r} b' \vec{e}_r \]

\( \times \) Force

\( \times \) force in the radial direction

\( \times \) hap in \( x-y \) plane for \( g_{\tau} w > 0 \)

but \( B = 0 \) at \( x = y = 0 \)

\( \Rightarrow \) Majorana clock

(2) Add homogeneous field along \( z \)-axis

for example by adding Wilson loops:

\[ B_0 = \vec{B}_0 \cdot \vec{e}_z \]

\[ |\vec{B}| = \left\{ B_0^2 + (b' r)^2 \right\}^{1/2} \approx B_0 + \frac{b' r^2}{2 B_0} \]

\[ \text{for } b' r \ll B_0 \]
\[ V(x) = V_0 + \frac{1}{2} m \omega_x^2 x^2 = \frac{g^T u_T}{h^3} \left( B_0 + \frac{b_1 x^2}{2 B_0} \right) \]

\[ \omega_x^2 = \sqrt{\frac{g^T u_T}{m B_0}} b_1 \]

*The smaller \( B_0 \) the larger the trap frequency*

**3) Confinement in \( \pm \)-direction**

*Add pinch coils*

*Two coils*

*Current flow in the same direction*

*But distance between coils much larger than Larmor radius*

\[ \Rightarrow B \text{-field has a minimum} \]

![Diagram](image)

\[ B_{\text{pinch}}(x) = B_{\text{pinch}}(0) + \frac{d^2 B_0}{dx^2} \frac{x^2}{2} \]

*Field curvature*

\[ \Rightarrow \text{confinement in axial (} \pm \text{-direction)} \]
Some estimates

\[ b^1 \propto \frac{3}{\mu} \]

\[ b_0 = 3 \times 10^{-4} \text{ } \text{I} \]

\[ b_{ii} = \frac{\partial^2 \bar{3}}{\partial t^2} \approx 300 \frac{1}{\mu^2} \]

\[ \Rightarrow \frac{\omega_i}{2\pi} = 15 \text{ mT} \quad \text{for Na} \]

\[ \omega_r / 2\pi = 250 \text{ mT} \]