About Synthetic Quantum Magnetism in Optical Lattices

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Outline

- Magnetic fields in superconducting systems: Meissner-Ochsenfeld-Effect
- Magnetic fields and lattice systems
- Real space alkali ladders (Munich)
- Synthetic dimensions (Florence)
- Strongly interacting systems: Gaped and gapless M- and V-phases
- Not so weak but less than strong: A surprising world
- Weak interactions: The Josephson-limit
- Swimming against the tide
- Density dependent magnetic fields
Magnetic fields in superconducting systems

- 1933 Meissner and Ochsenfeld: perfect diamagnetism of superconductors
- Screening by thin layer of surface currents
- For $H > H_C$ external field can penetrate through vortices (type II)
Peierls substitution

Landau gauge $\mathbf{A} = (0, B x, 0)$ (homogeneous B-field). Transition amplitude along $x$ gains $y$-dependence:

$$J_x \rightarrow J_x e^{i a^2 B \frac{e}{\hbar} y} = J_x e^{i \Phi y} = J_x e^{i 2\pi \alpha y}$$

$$\mathcal{H} = -J_x \sum_{x,y} e^{i \Phi y} b_{x,y}^\dagger b_{x+1,y} - J_y \sum_{x,y} b_{x,y}^\dagger b_{x,y+1} + h.c.$$
The simplest case: two-leg ladder

\[ \mathcal{H} = -J \sum_r (b_{1,r}^\dagger b_{1,r+1} + b_{2,r}^\dagger b_{2,r+1}) - J_{\perp} \sum_r e^{i\phi_r} b_{1,r}^\dagger b_{2,r} + H.c. \]

- interactions \( \frac{U}{2} \sum_r n_r (n_r - 1) \)
- total current along leg \( \alpha \)

\[ j^\alpha = -\frac{i}{2} \left\langle \sum_x b_{\alpha,x}^\dagger b_{\alpha,x+1} - H.c \right\rangle \]

- gauge invariant chiral current

\[ J_c = j^1 - j^2 = \partial_\phi \langle \mathcal{H} \rangle \]
Fluxladders with in alkali bosons


- Local optical potential by two far-detuned running-wave beams realizes spatial dependent phase of the shaking $\Phi_{x,y} = \mathbf{q} \cdot \mathbf{r}/2$

$$V_0 \cos^2(\omega t/2 + \mathbf{q} \cdot \mathbf{r}/2)$$

- tilt $\Delta = \omega$ given by superlattice potential

- with $\tilde{J}_x = J_x J_1 (2 \frac{V_0}{\omega} \sin(\Phi_{x,y} - \Phi_{x+1,y})/2)$
  and $\tilde{J}_y = J_y J_0 (\ldots)$

$$H_{\text{eff}} = - \tilde{J}_x \sum_y e^{i \Phi_{x,y}} b_{1,y}^\dagger b_{2,y}$$

$$- \tilde{J}_y \sum_{x,y} b_{x,y}^\dagger b_{x,y+1} + H.c.$$ 

- Realization with $^{87}\text{Rb}$ of flux $\phi = \pi/2$ in (Bloch) and $\phi = \pi$ (Ketterle) per plaquette
Ladders in synthetic dimensions


- coupling of internal spin states via two-photon Raman transition: $\Omega e^{i\phi_x}$
- finite-sized system with sharp and addressable edges
- $^{173}\text{Yb}$, $\phi \simeq 0.37\pi$, $I = 5/2 \leftrightarrow$ up to 6 legs
**Single-particle picture**

*Atala et al., Nature Physics 10, 588-593 (2014)*

- Non-interacting BH-model on a ladder with fluxes $\Phi$. 

![Graphical representation of the single-particle picture with Meissner and Vortex phases](image)
Strongly interacting bosons: $U \rightarrow \infty$


- Gaped and gapless Vortex and Meissner phases
- Totally gapped phase also at $\rho = 1/4$ (Rung-Singlet-Néel)
- upper V-SF phase vanishes for large $J_{\perp}$
- No Vortex-lattice phase!
Two leg Bosonic Flux Ladders

Vortex lattice $\rho_V = \frac{1}{2}$

Vortex superfluid

Vortex lattice $\rho_V = \frac{1}{3}$

Vortex superfluid

Meissner phase $\rho_V = 0$
Finite $U \sim J$: Phase diagram

Vortexlattice phase at $\rho_V = 1/3$ vortex density
Weak interactions and high fillings: “The Josephson-limit”

- \( \rho \gg 1, \ U \to 0 \)
- On each site \( x \) BEC with defined local phase \( \theta_x \)
- \( a_x^\dagger \sim \sqrt{\rho} e^{i\theta_x} \)
- Map to frustrated classical spin model
  \[
  \mathcal{H}/\rho \simeq -J \sum_{x,y} \cos(\theta_{y,x+1} - \theta_{y,x} - \phi) \\
  - J_\perp \sum_x \cos(\theta_{1,x} - \theta_{2,x})
  \]
- At \( T = 0 \) devil's-staircase-like phase diagram, Vortex-lattice phases at each rational \( \rho_V = p/q \)

Finite temperature phase diagram \( k_B T / J = 5 \cdot 10^{-3} \)
Comparison with experiments

- Vortex-lattice phases for 5-10 times smaller $T$
- Surprising observation: negative $J_c$ 😲!
Summary

- Various recent experimental realizations of synthetic magnetic fields in optical lattices (Munich, Florence, NIST, ...)
- “Simplest case” ladder models offer surprisingly rich physics
- Gaped and gapless M- and V-phases in simplest hardcode boson (and fermion) systems
- VL and BL phases for finite interacting bosonic systems
- VL phases may be characterized by novel current reversal phenomenon
- The Anyon Hubbard model in one dimensional optical lattices
- Density dependent magnetic fields: Feedback on the gauge field
Thank you for your attention!