Dipolar quantum gases

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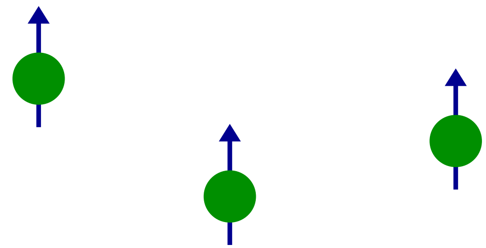
Outline

- Experiments with ultracold dipolar particles
- Scattering problem. Dipolar bosons
- Dipolar BEC. Stability problem
- Dipolar Fermi gas. Scattering problem
- Superfluid pairing
- Several ideas

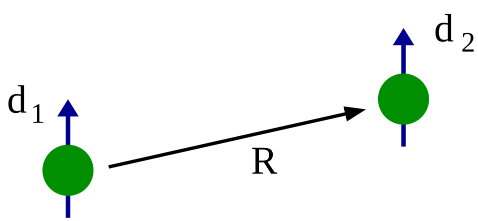
Hannover, April 11, 2014

Novel object - Dipolar gas

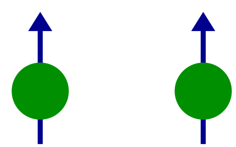
Polar molecules or atoms with a large magnetic moment



Dipole-dipole interaction $V_d = \dfrac{\vec{d}_1\vec{d}_2R^2 - 3(\vec{d}_1\vec{R})(\vec{d}_2\vec{R})}{R^5} \sim \dfrac{1}{R^3}$



long-range, anisotropic



repulsion

attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules d from $0.6\,D$ for KRb to $5.5\,D$ for LiCs

Atoms with large μ

Remarkable experiments with Cr atoms ($\mu = 6\mu_B \Rightarrow d \approx 0.05$ D)

T. Pfau group (Stuttgart)

Effects of the dipole-dipole interaction in the dynamics

Stability diagram of trapped dipolar BEC

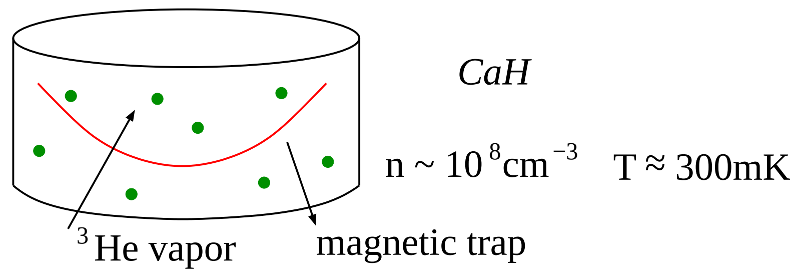
Spinor physics in chromium experiments at Villetaneuse, B. Laburthe-Tolra

Now disprosium ($\mu = 10\mu_B$, (B. Lev))

and erbium ($\mu = 7\mu_B$, (F. Ferlaino)) are in the game

Polar molecules. Creation of ultracold clouds

- Buffer gas cooling (Harvard, J. Doyle)



- Stark deceleration (Meijer, Berlin; JILA)

  H_3, ND_3, CO, etc. ; $T \sim 1mK$ and low density

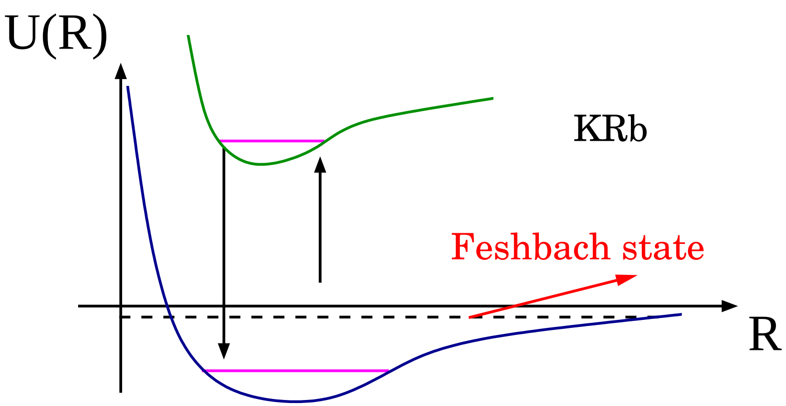
- Optical collisions (photoassociation, D. DMille group, JILA, elsewhere)

Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb fermionic molecules to the ground rovibrational state

JILA, D. Jin, J. Ye groups



$$n \sim 10^{12} - 10^{13}\ \text{cm}^{-3}$$
$$T \approx 200nK\ \sim E_F$$

Ground-state LiCs molecules at Heidelberg

Ground-state RbCs molecules in Innsbruck

Ground-state KRb bosonic molecules in Tokyo

xperiments with NaK (MIT, Munich, Hannover, Trento) and KCs (Innsbruck) molecule

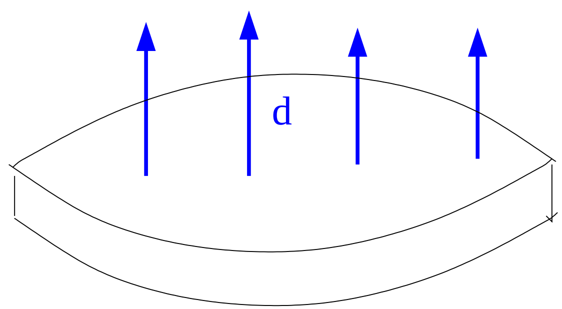
Ultracold chemistry

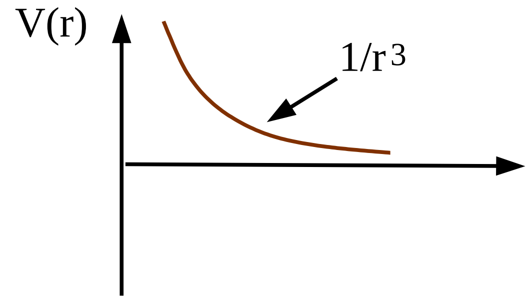
Ultracold chemical reactions $\mathrm{KRb} + \mathrm{KRb} \Rightarrow \mathrm{K_2} + \mathrm{Rb_2}$

New trends in ultracold chemistry

Suppress instability $\rightarrow$ induce intermolecular repulsion

For example, 2D geometry with dipoles perpendicular to the plane





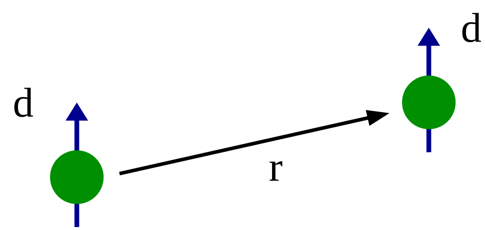
Reduction of the decay rate by 2 orders of magnitude at JILA

Select non-reactive molecules, like NaK, KCs, RbCs

Theoretical studies

- Innsbruck group (P. Zoller, G. Pupillo, M.A. Baranov et al). Large variety of proposals including bilayer systems, Rydberg atoms etc.
- Trento group (S. Stringari et al). Excitation modes etc
- Harvard group (E. Demler, M. Lukin et al). Multilayer systems etc
- Hannover group (L. Santos et al). Spinor and dipolar systems
- Tokyo group (M. Ueda et al) Spinor and dipolar systems
- Cambridge group (N.R. Cooper, Jesper Levinsen). Novel states
- Rice group (H. Pu et al). Excitations and stability etc
- Maryland group (S. Das Sarma et al) Fermi liquid behavior etc
- Taipei group (D.-W. Wang et al)
- Barcelona group (M. Lewenstein et al)

Dipole-diple scattering



$$V_d = \frac{d^2}{r^3} \underbrace{\left(1 - 3\cos^2\theta_{rd}\right)}_{\sim Y_{20}(\theta_{rd})}$$

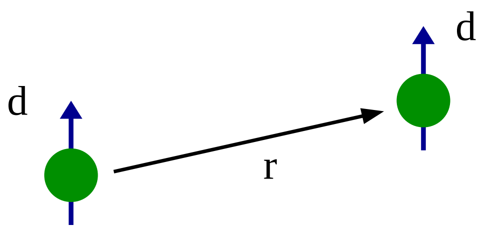
Wave function of the relative motion

$$\psi_{in} \to \sum_{l,m} \psi_{kl}(r) i^l Y^*_{lm}(\theta_{kd}, \varphi_{kd}) Y_{lm}(\theta_{rd}, \varphi_{rd})$$
$$\psi_{out} \to \sum_{l',m'} \psi_{kl'}(r) i^{l'} Y^*_{l'm'}(\theta_{kd}, \varphi_{kd}) Y_{l'm'}(\theta_{rd}, \varphi_{rd})$$

Scattering matrix $\sim \int Y^*_{lm} Y_{l'm'} Y_{20} d\Omega_{rd}$

V_d couples all even l, and all odd l, but even and odd l are decoupled from each other

Radius of the dipole-dipole interaction



$$\left(-\frac{\hbar^2}{m}\Delta + V_d(\vec{r})\right)\psi(\vec{r}) = \frac{\hbar^2 k^2}{m}\psi(\vec{r})$$

$$\frac{\hbar^2}{m r_*^2} = \frac{d^2}{r_*^3} \Rightarrow r_* \approx \frac{m d^2}{\hbar^2}$$

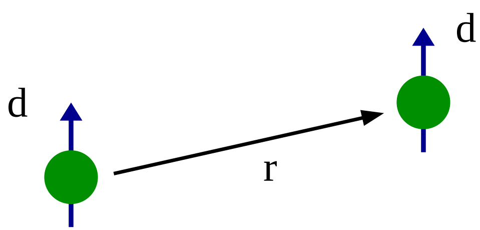
$r \gg r_*$ $\rightarrow$ free relative motion

$r_* \sim 10^6 \div 10^3 a_0$ polar molecules

$r_* \approx 50 a_0 \rightarrow$ chromium atoms

$$k r_* \ll 1 \quad \rightarrow \underbrace{\text{Ultracold limit}}_{T \ll 1mK \quad \text{for} \quad \text{Cr}}$$

Scattering amplitude I



$$V(\vec{r}) = \mathcal{U}(\vec{r}) + V_d(\vec{r})$$

$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3 r$$

Ultracold limit $kr_* \ll 1$

$$V_d = 0 \Rightarrow f = g = \frac{4\pi\hbar^2}{m} a$$

What V_d does?

$$k = 0 \rightarrow g = \int \psi^*_0(\vec{r})(\mathcal{U}(\vec{r}) + V_d(\vec{r})) d^3 r = \text{const}; \qquad r \lesssim r_*$$

g may depend on d and comes from all even l

Scattering amplitude II

$$k \neq 0$$

$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3 r$$

$$r \lesssim r_* \rightarrow \text{put } k = 0 \rightarrow g$$

$$r \gg r_* \rightarrow \psi_{k_i} = e^{i\vec{k}_i \vec{r}}$$

$$f = \int V_d(\vec{r}) e^{i\vec{q}\vec{r}} d^3 r \longrightarrow \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1); \ \vec{q} = \vec{k}_f - \vec{k}_i$$

$$f = g + \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1)$$

Dipolar BEC I

Uniform gas $g > 4\pi d^2/3 \to f > 0 \Rightarrow$ stable BEC

$$H = \int d^3 \Big[\psi^\dagger(\vec{r}) \left(-\frac{\hbar^2}{2m}\Delta\right)\psi(\vec{r}) + \tfrac{1}{2} g \psi^\dagger(\vec{r})\psi^\dagger(\vec{r})\psi(\vec{r})\psi(\vec{r})$$
$$+\tfrac{1}{2}\int d^3 r' \psi^\dagger(\vec{r})\psi^\dagger(\vec{r}')V_d(\vec{r}-\vec{r}')\psi(\vec{r}')\psi(\vec{r})\Big]$$

Bogoliubov approach $\psi = \psi_0 + \delta\Psi \to$ biliniear Hamiltonian

$$H_B = \frac{N^2}{2V} g + \sum_k \left[\frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + n\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right) a_k^\dagger a_k\right.$$
$$\left.\frac{n}{2}\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right)\left(a_k^\dagger a_{-k}^\dagger + a_k a_{-k}\right)\right]$$

Dipolar BEC II

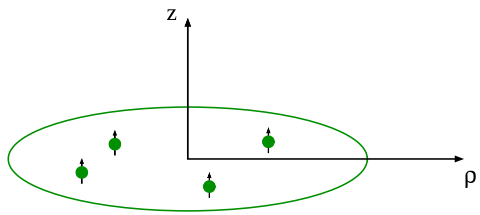
Excitation spectrum

$$\epsilon_k = \sqrt{E_k^2 + 2E_k n \left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right)}$$

$g > \frac{4\pi d^2}{3} \to$ dynamically stable BEC

$g < \frac{4\pi d^2}{3} \to$ complex frequencies at small k
and $\cos^2\theta_k < \frac{1}{3} \to$ collapse

Trapped dipolar BEC



Cylindrical trap

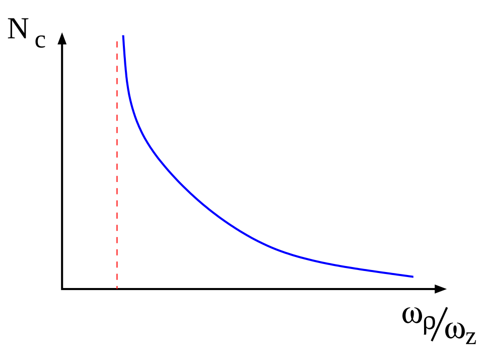
$V_h = \frac{m}{2}\left(\omega_\rho^2 \rho^2 + \omega_z^2 z^2\right)$

Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\Delta + V_h(\vec{r}) + g\psi_0^2 + \int \psi_0(\vec{r}')^2 V_d(\vec{r}-\vec{r}')d^3r'\right]\psi_0(\vec{r}) = \mu\psi_0(\vec{r})$$

Important quantity

$$V_{eff} = g\int \psi_0^4(\vec{r})d^3r + \int \psi_0^2(\vec{r}')V_d(\vec{r}-\vec{r}')\psi_0^2(\vec{r})d^3r d^3r'$$



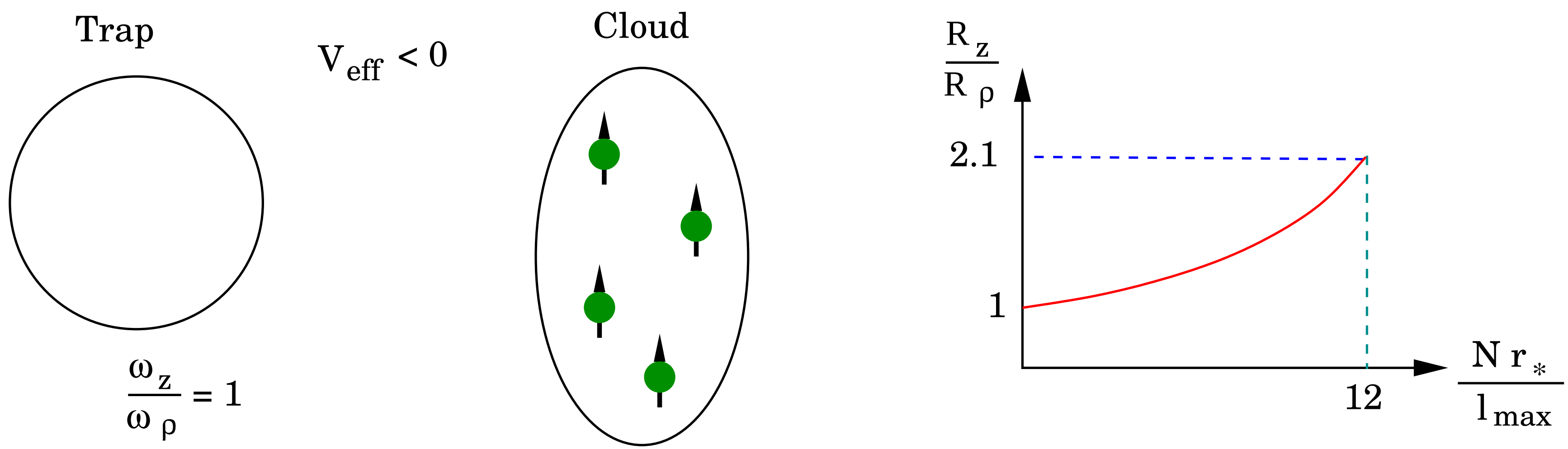
$V_{eff} > 0$ or $V_{eff} < 0$ and $|V| < \hbar\omega$

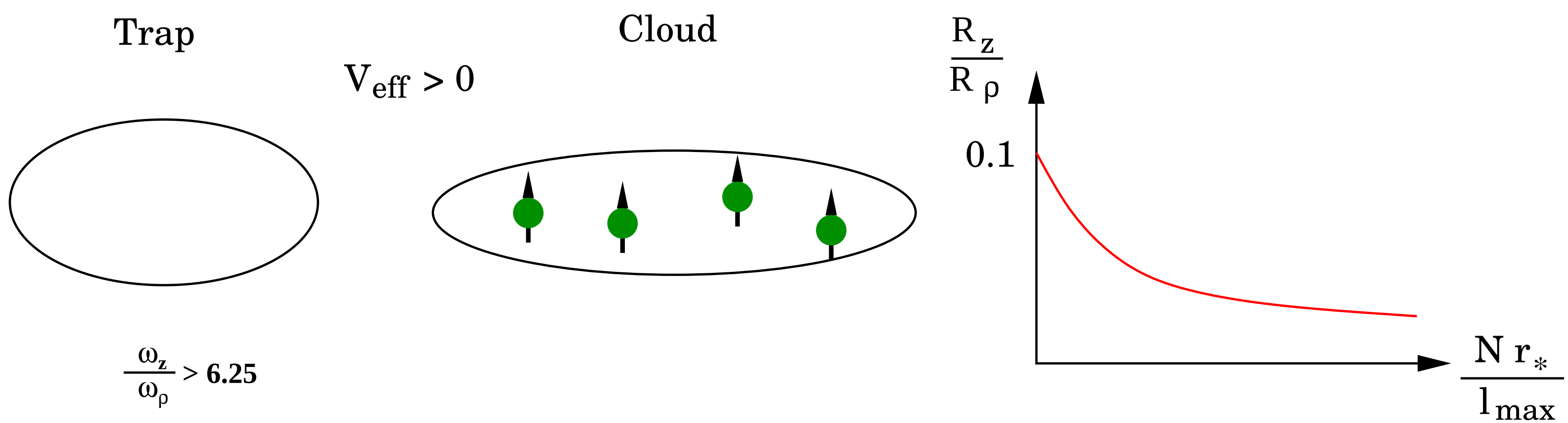
$g = 0 \to N < N_c \to$ suppressed

low k instability

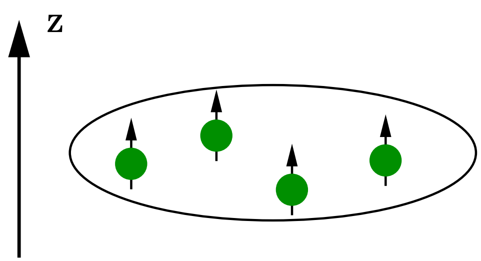
(Santos et.al, 2000)

It is sufficient?





Stability problem



Dipolar BEC

$$\langle V_d \rangle = \int n_0(\vec{r}')V_d(\vec{r}' - \vec{r})d^3r = -d^2\frac{\partial^2}{\partial z^2}\int \frac{n_0(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3r$$

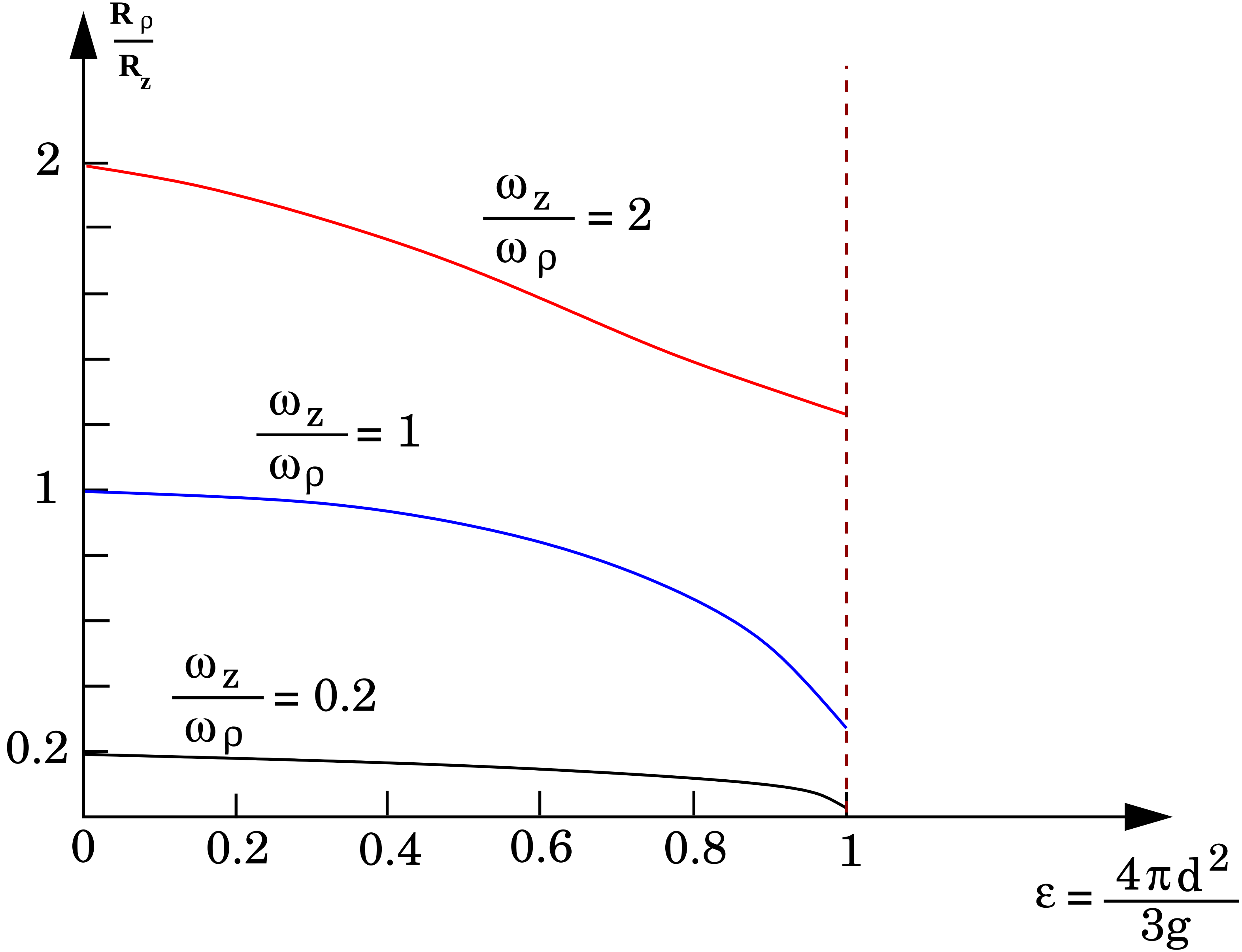
$$V_d = -d^2\frac{\partial^2}{\partial z^2}\frac{1}{|\vec{r}-\vec{r}'|} - \frac{4\pi d^2}{3}\delta(\vec{r} - \vec{r}')$$

Large $N \Rightarrow$ Thomas-Fermi BEC

$$n_0 = n_{0\,max}\left(1 - \frac{z^2}{R_z^2} - \frac{\rho^2}{R_\rho^2}\right)$$ Eberlein et. al (2005)

$g > \frac{4\pi d^2}{3} \rightarrow$ stable at any N

Example



$\frac{R_\rho}{R_z}$
2
1
0.2
$\frac{\omega_z}{\omega_\rho} = 2$
$\frac{\omega_z}{\omega_\rho} = 1$
$\frac{\omega_z}{\omega_\rho} = 0.2$
0
0.2
0.4
0.6
0.8
1
$\varepsilon = \frac{4\pi d^2}{3g}$

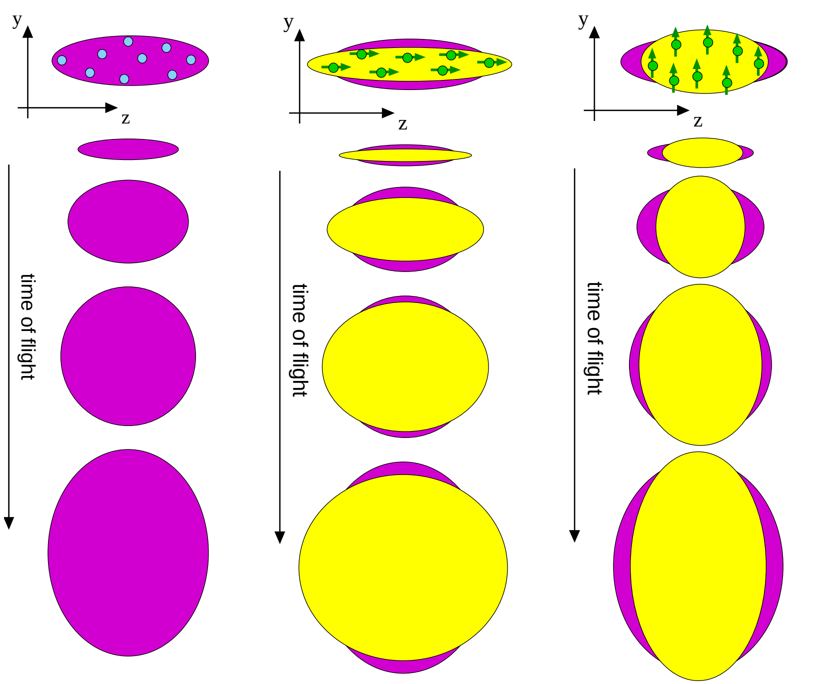
Experiment with Cr

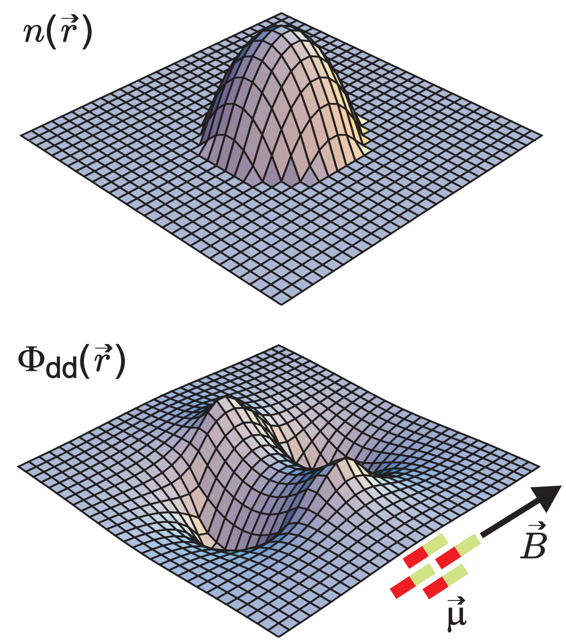
$$g > \frac{4\pi d^2}{3}$$

$(\mu = 6\mu_B!)$

(T. Pfau, Stuttgart) BEC ($n \sim 10^{14} cm^{-3}$)

effect of the dipole-dipole interaction (small)



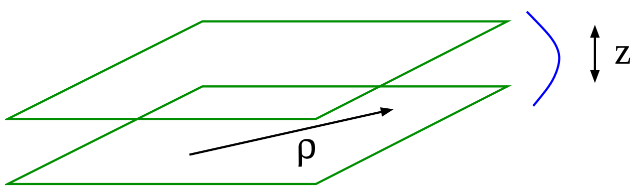


Pancake dipolar BEC

$$g < g_d = \frac{4\pi d^2}{3} \quad ?$$

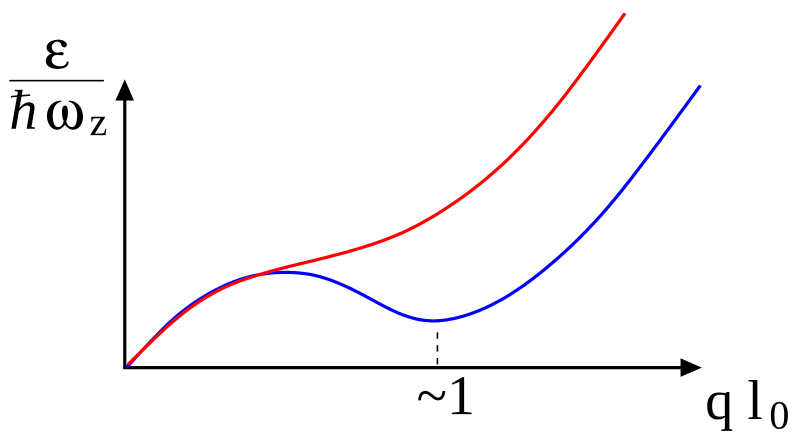
Thomas-Fermi in the z direction

Extreme pancake ($\omega_\rho = 0$)



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$V_d + g$ (short-range) $\quad g > 0$

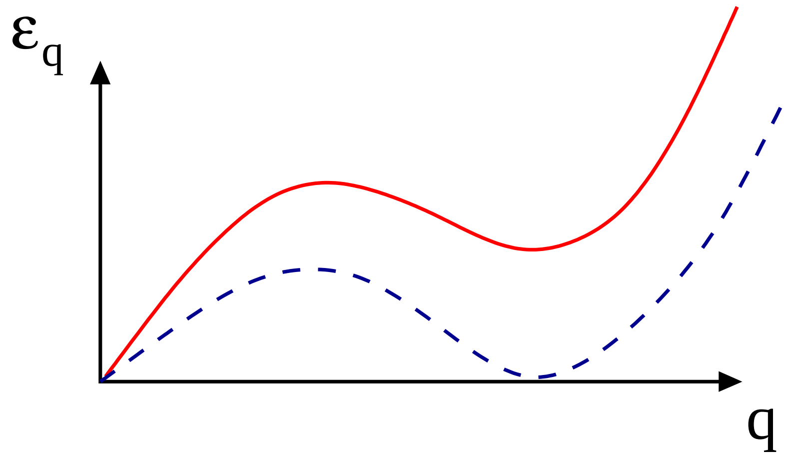


$g/g_d = 1.06 \qquad \mu/\hbar\omega_z = 46$

$g/g_d = 0.94 \qquad \mu/\hbar\omega_z = 53$

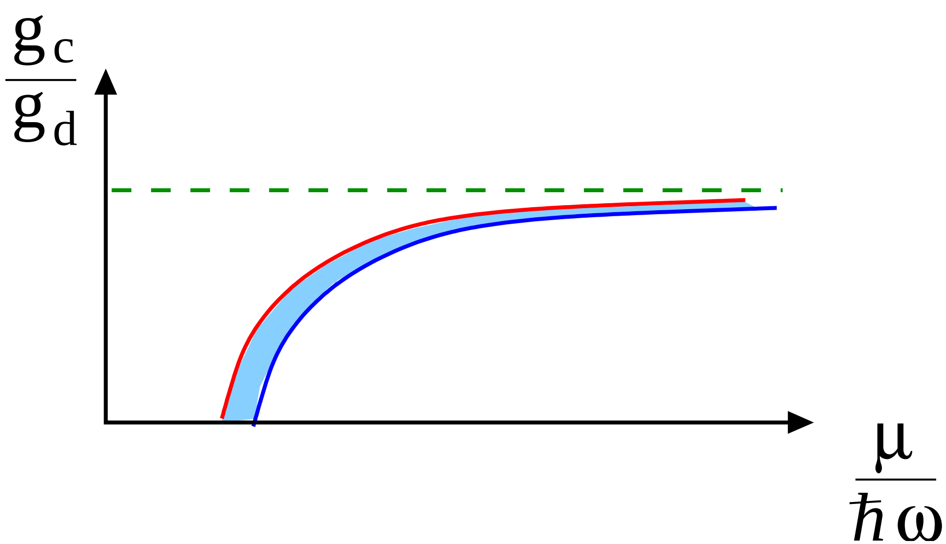
Roton structure $\Rightarrow$ decrease of the interaction amplitude

Roton-maxon structure



Roton minimum can be but at zero

Instability!

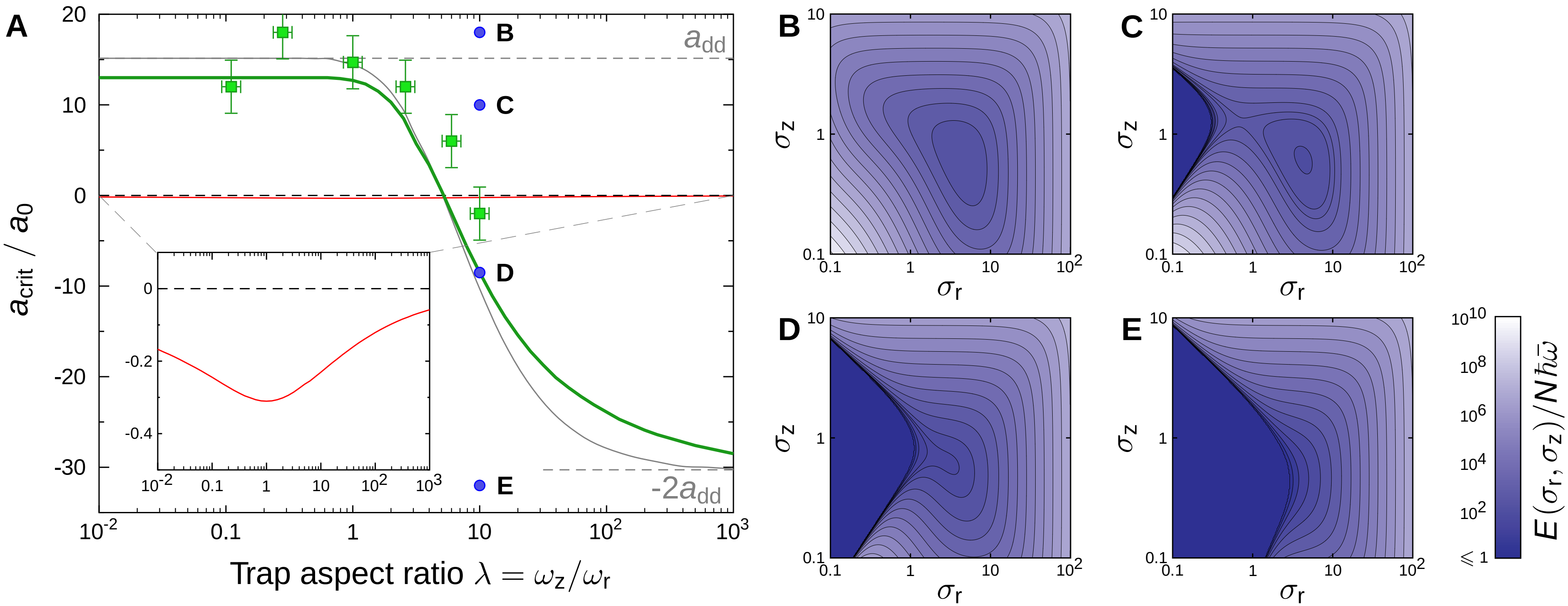


rotonization

instability

(L. Santos et al., 2003)

Stuttgart experiment

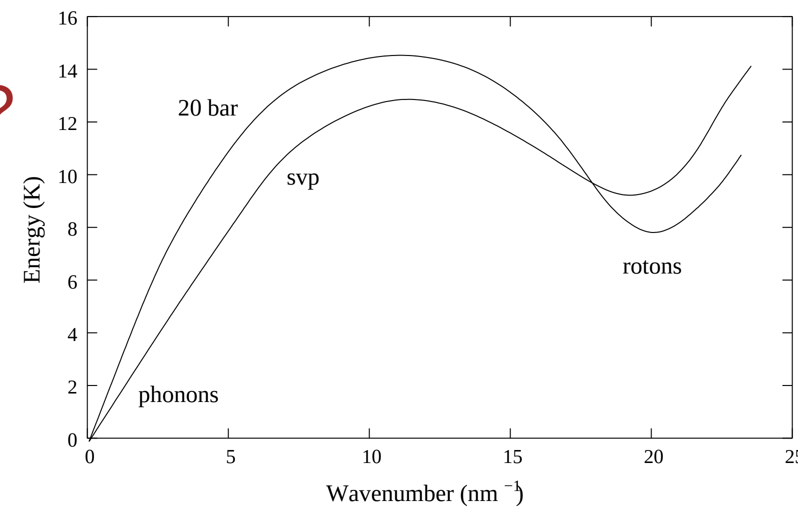


Ideas from superfluid ^4He

Why the roton-maxon structure?

R. Feynman

$$\epsilon_q = \frac{q^2}{2mS(q,\epsilon)}$$



How to put the roton minimum higher (*) or lower (**)?

What happens? (S.Balibar, P. Nozieres, L. Pitaevskii)

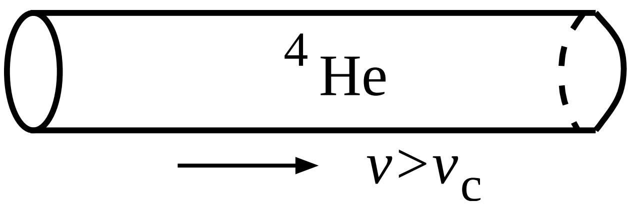
(*) negative pressure (acoustic pulses)
(**) increase the pressure

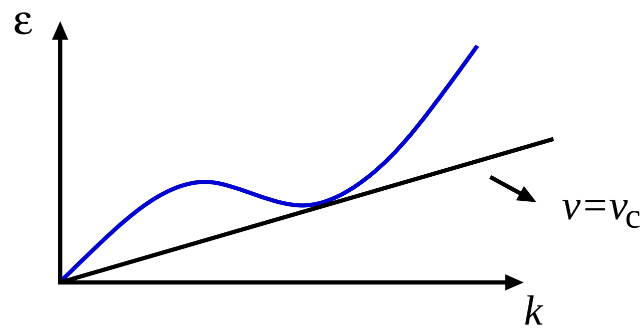
Metastable liquid

(**) $\Rightarrow$ supersolid (density wave), loss of superfluidity, or?

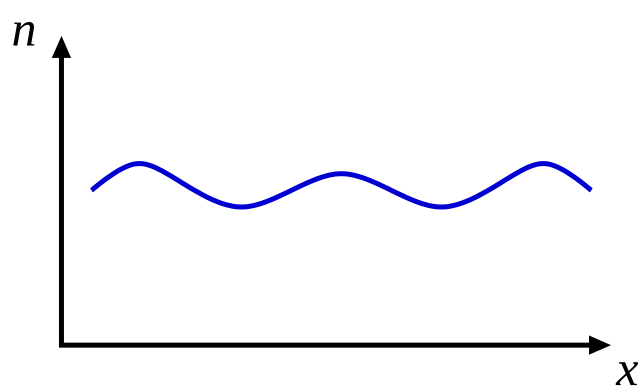
Prehistory

L.P. Pitaevskii(1981)

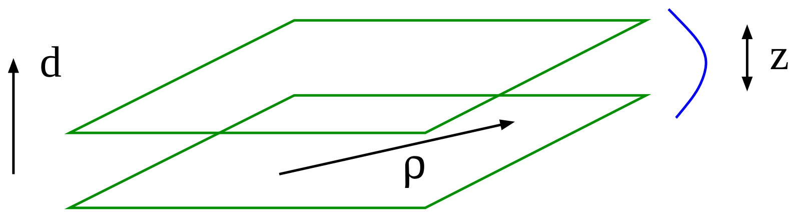




$v > v_c \quad \Rightarrow$ Density wave



Quasi2D dipolar BEC at $T = 0$



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{\frac{-z^2}{2l_0^2}\right\}$$

short range interaction (g) + dipole-dipole

Consider $0 < g \ll \dfrac{4\pi d^2}{3}$. Then, for $qr_* \ll 1$

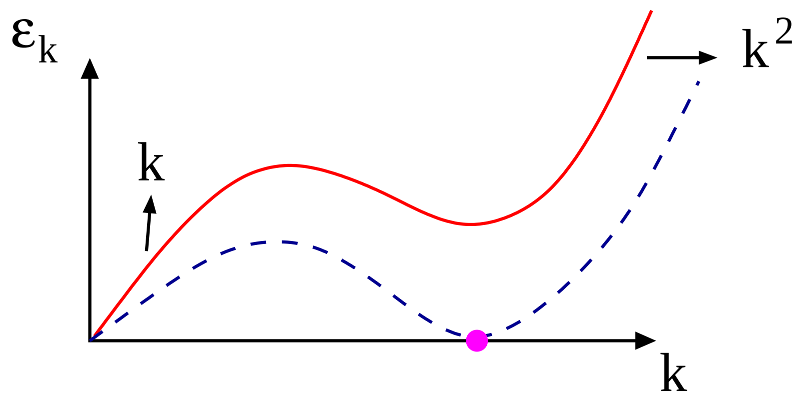
$$V_{\vec{q}\vec{p}} = g(1 - C|\vec{q} - \vec{p}|)$$

where

$$C = \frac{2\pi d^2}{g}$$

Spectrum

$$\hat{H} = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a^\dagger_{\vec{k}} a_{\vec{k}} + \frac{g}{2} \sum_{\vec{k},\vec{q},\vec{p}} (1 - C|\vec{q} - \vec{p}|) a^\dagger_{\vec{k}+\vec{q}} a^\dagger_{\vec{k}-\vec{q}} a_{\vec{k}+\vec{p}} a_{\vec{k}-\vec{p}}$$



$$\epsilon_k^2 = E_k^2 + 2\mu E_k (1 - Ck)$$

$$k_r = \frac{3}{2}\left(1 + \sqrt{\frac{C^2}{\xi^4} - \frac{8}{9\xi^2}}\right) \quad \xi = \frac{\hbar}{mng}.$$

Rotonization

$$\xi \geq C \geq \frac{\sqrt{8}}{3}\xi$$

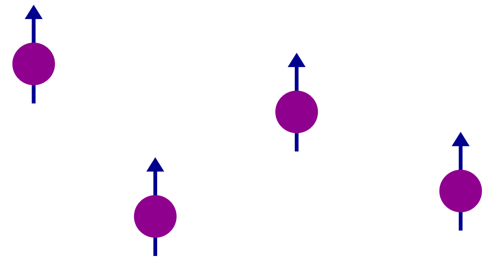
The roton minimum touches zero for

$$C = \xi \Rightarrow \quad k_r = \frac{2C}{\xi}$$

For $C > \xi$ we have collapse. No stable supersolid state

Pedri/Shlyapnikov; Cooper/Komineas (2007)

Dipolar Fermi gas



What does th dipole-dipole interaction do in a Fermi gas?

- Single component gas

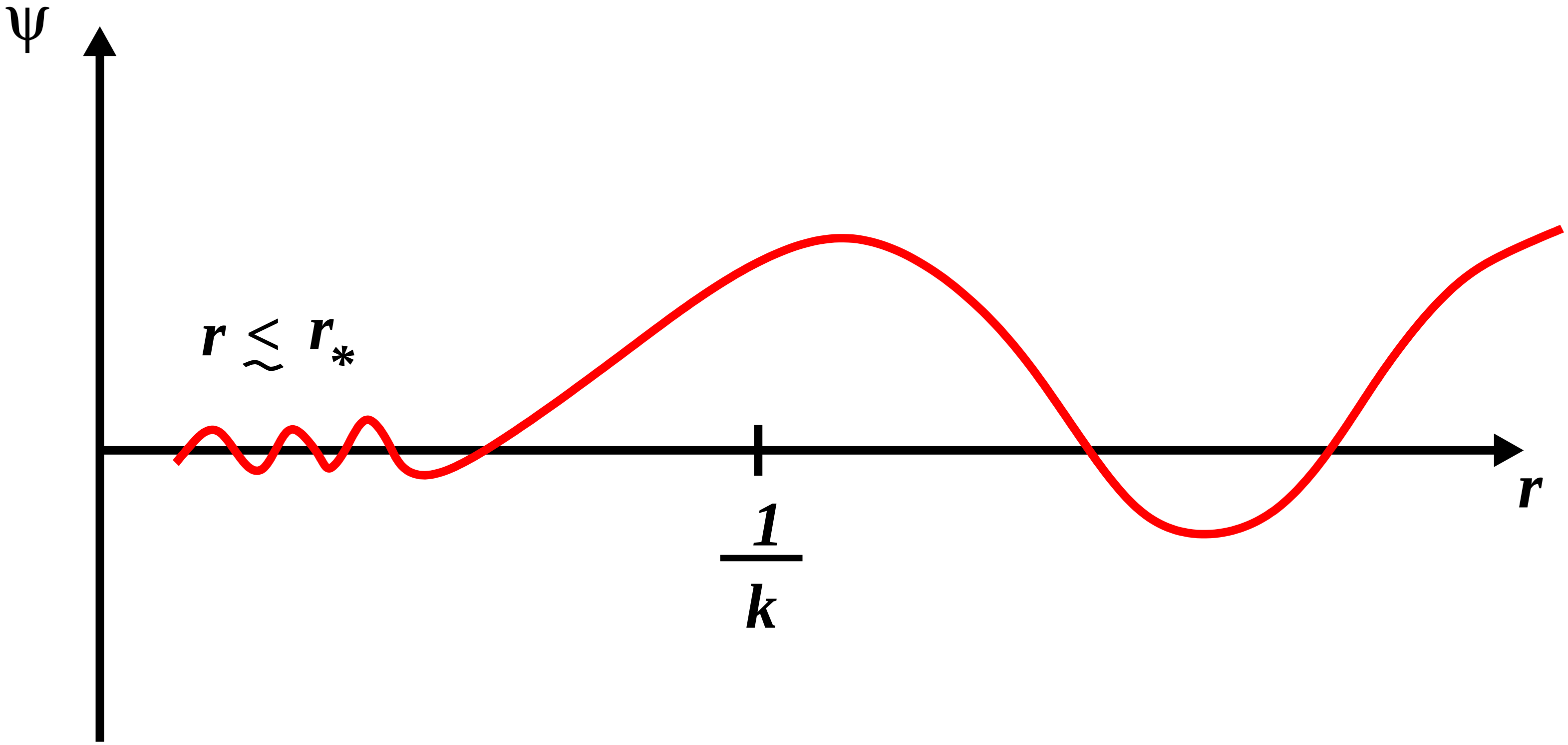
The dipole-dipole scattering amplitude is independent of $|k|$ at any orbital angular momenta alowed by the selection rules.

Long-range cotribution $\sim d^2$

$$\underbrace{\text{Short-range cotribution} \sim k^2}_{\text{omit}}$$

Universal result for f

Physical picture. Stability



nd^2 significantly larger than $E_F = \dfrac{(6\pi^2 n)^{2/3}}{2m}$,

which is $k_F r_*$ significantly larger than 1, leads to collapse

Odd-l scattering amplitude

$$f = \frac{1}{2}\int \left(e^{i\vec{k}_i\vec{r}} - e^{-i\vec{k}_i\vec{r}}\right) V_d(\vec{r}) \left(e^{-i\vec{k}_f\vec{r}} - e^{i\vec{k}_f\vec{r}}\right) d^3r =$$
$$= 4\pi d^2(\cos^2\theta_{dq_-} - \cos^2\theta_{dq_+})$$

$$\vec{q}_\pm = \vec{k}_i - \vec{k}_f$$

$$e^{i\vec{k}\vec{r}} = 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} j_l(kr) i^l Y_{lm}(\hat{r}) Y^*_{lm}(\hat{k})$$

Partial amplitude $\quad f(l_i m_i; l_f m_f)$

$$f(10;10) = -\frac{6d^2}{5}\cos\theta_{dk_i}\cos\theta_{dk_f}$$

Superfluid p-wave pairing

$$nd^2 \ll E_F$$

Analog of $a \to \sim \frac{md^2}{\hbar^2}$ $(r_*!)$

$$\Delta = g\langle\psi_\uparrow\psi_\downarrow\rangle \sim E_F \exp\left(-\frac{1}{\lambda}\right); \; \lambda \sim \frac{1}{k_F r_*};$$

$$\Delta \propto E_F \exp\left\{-\frac{\pi E_F}{12nd^2}\right\}$$

Cooper pairs are superpositions

of all odd angular momenta (for $m_l = 0$)

Transition temperature

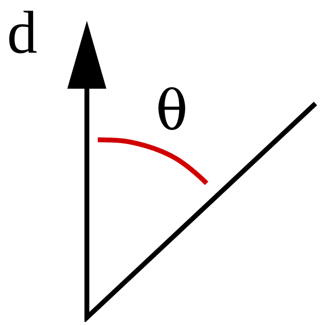
$$nd^2 \ll E_F$$

$$T_c = 1.44 E_F \exp\left\{-\frac{\pi E_F}{12 n d^2}\right\}$$

Baranov et.al (2002)

GM correction included

$\Delta \to$ anisotropic $\propto \sin\left(\frac{\pi}{2}\cos\theta\right)$



- Maximum in the direction of the dipoles.
- Vanishes in the direction perpendicular to the dipoles

Distiguished features

Anisotropic gap $\rightarrow$ gapless excitations

in the direction perpendicular to

the dipoles. Damping rates etc.

Anisotropy $\rightarrow$ different from p-wave

superfluid B and A phases of ^{3}He. In B

Δ is isotropic, and in A it vanishes only at

2 points on the Fermi shpere ($\theta = 0$ and $\theta = \pi$)

Value of T_c

$$d \sim 0.1 \div 1\ \mathcal{D}$$

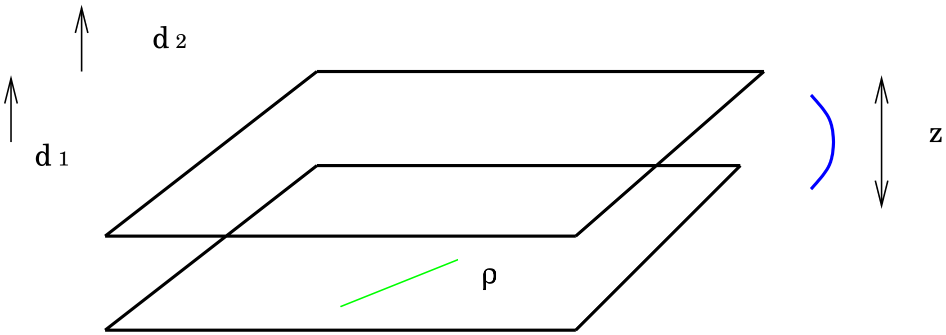
NaK $\Rightarrow$ $d = 2.7\mathcal{D}$ and can be made $0.5\mathcal{D}$ ($r_* = 2500\mathring{A}$) in a certain electric field

$\frac{T_c}{E_F} \to 0.025$ at $n \to 6 \times 10^{12}\mathrm{cm}^{-3}$ ($E_F \approx 400$ nK

($T_c \to 10nK$)

One easily achieves the strongly interacting regime

2D dipolar Fermi gas



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{\frac{-z^2}{2l_0^2}\right\}$$

2-component Fermi gas (↑ and ↓). Short range $g > 0$

$$H = \sum_k \frac{\hbar^2 k^2}{2m}\left(a^\dagger_{k\downarrow} a_{k\downarrow} + a^\dagger_{k\uparrow} a_{k\uparrow}\right) +$$
$$+|g| \sum_{k,p,q} (1 - C|\vec{q} - \vec{p}|)\, a^\dagger_{k+q\uparrow} a^\dagger_{k-q\downarrow} a_{k-p\downarrow} a_{k+p\uparrow}$$

$$C = \frac{2\pi d^2}{|g|}$$

Interesting problem

s-wave interaction on the Fermi surface

$$|g|(1 - 4Ck_F/\pi)$$

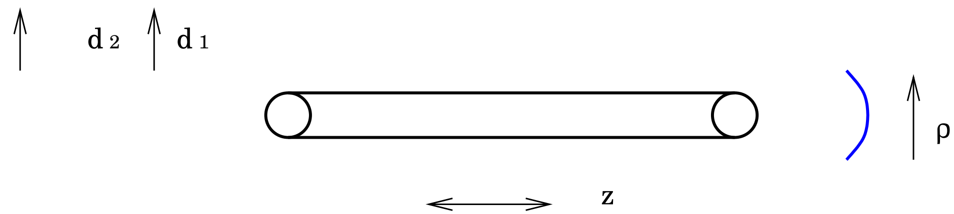
$k_F C > \pi/4 \to 8d^2 k_F > |g| \to$ superfluidity

$k_F C < \pi/4 \to 8d^2 k_F < |g| \to$ no superfluidity

Quantum transition to a normal state with decreasing density

Superfluid pairing for tilted dipoles $\to$ Baranov/Sieberer (2011)

1D dipolar Fermi gas



$$l_0 = \left(\frac{\hbar}{m\omega_\rho}\right)^{1/2}$$

$$\varphi_0(\rho) = \frac{1}{\pi^{1/2} l_0} \exp\left\{\frac{-\rho^2}{2l_0^2}\right\}$$

short range $\quad g > 0$

$$H = \sum_k \frac{\hbar^2 k^2}{2m} \left(a^\dagger_{k\downarrow} a_{k\downarrow} + a^\dagger_{k\uparrow} a_{k\uparrow}\right) +$$

$$+|g| \sum_{k,p,q} \left(1 + B|\vec{q} - \vec{p}|^2 \ln(|\vec{q} - \vec{p}| l_0)\right) a^\dagger_{k+q\uparrow} a^\dagger_{k-q\downarrow} a_{k+p\downarrow} a_{k-p\uparrow}$$

$$B = \frac{d^2}{|g|}$$

Quantum transition

Interaction at the Fermi points

$$g_{eff} = |g|[1 + 2B(k_F l_0)^2 \ln(k_F l_0)]$$

$g_{eff} < 0 \rightarrow$ superfluid

$g_{eff} > 0 \rightarrow$ ordinary Luttinger liquid

Quantum transition superfluid-Luttinger liquid with decreasing density

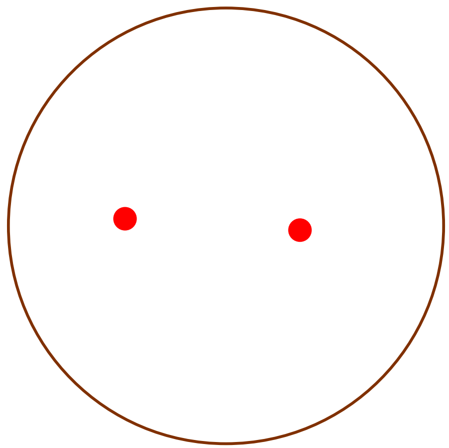
Are there novel many-body states?

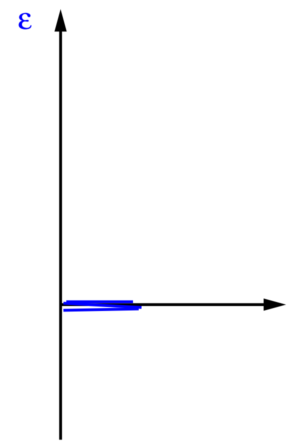
Does the dipole-dipole interaction lead to the emergence of novel phases for identical fermions?

Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)





The number of zero-energy states exponentially grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics $\Rightarrow$ Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

Topologically protected state for quantum information processing

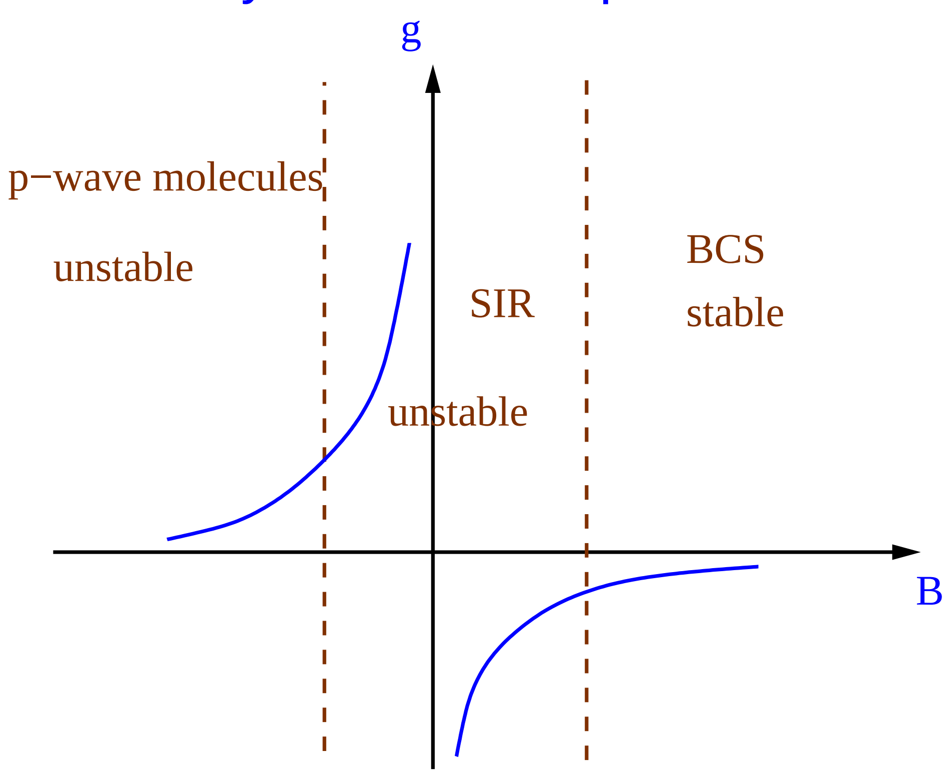
p-wave resonance for fermionic atoms

p-wave resonance Experiments at JILA, ENS, Melbourne, Tokyo, elsewhere

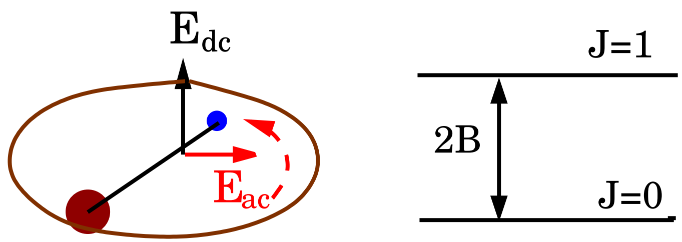
BCS $\Rightarrow$ $T_c \sim \exp\left(-\frac{1}{(k_F b)^2}\right)$ practically zero

Molecular and strongly interacting regimes $\Rightarrow$ rather high T_c, but collisional instability

Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio



RF-dressed polar molecules in 2D. Innsbruck idea



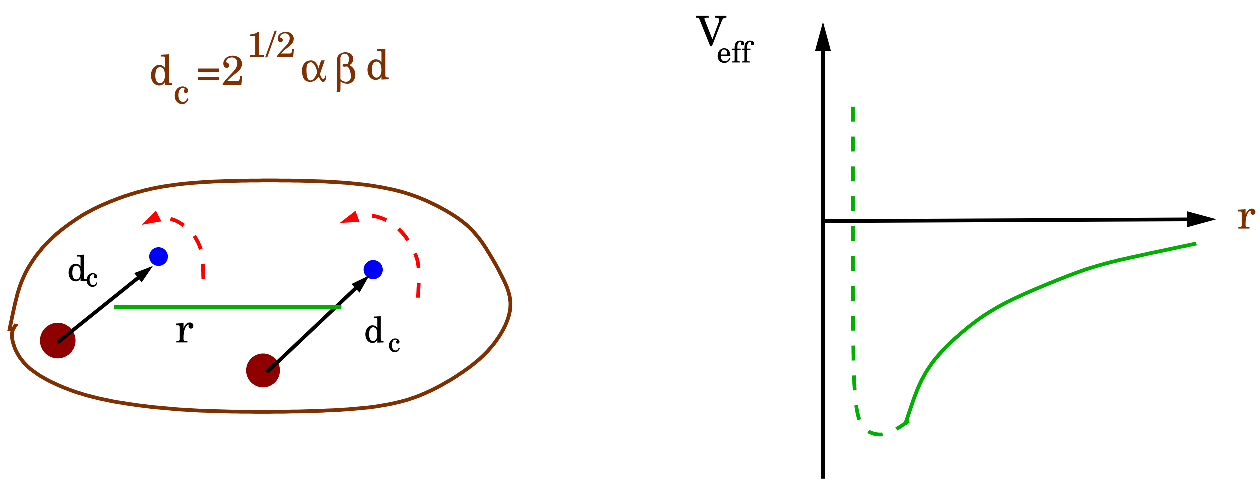
σ^+ polarized RF E_{ac}

+ constant $E_{dc} \longrightarrow 0$

Dressed states $|+\rangle = \alpha|0,0\rangle + \beta|1,1\rangle; \quad ,|-\rangle = \beta|0,0\rangle - \alpha|1,1\rangle$

$$\alpha = -\frac{A}{\sqrt{A^2+\Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2+\Omega^2}}; \quad A = \frac{1}{2}(\delta + \sqrt{\delta^2 + 4\Omega^2})$$

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency



Large $r \to \quad V_{eff} = \langle (1 - 3\cos^2\phi) \rangle \frac{d_c^2}{r^3} = -\frac{d_c^2}{2r^3}; \quad r_* = md_c^2/2\hbar^2$

Fermionic RFD molecules. Superfluid transition

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the p-wave scattering ($l = \pm 1$)

$$\hat{H} = \int d^2r\, \hat{\Psi}^\dagger(\mathbf{r})\{-(\hbar^2/2m)\Delta + \int d^2r' \hat{\Psi}^\dagger(\mathbf{r}')V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}') - \mu\}\hat{\Psi}(\mathbf{r})$$

$$\Delta(\mathbf{r}-\mathbf{r}') = \langle V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

Gap equation $\Delta(\mathbf{k}) = -\int \frac{d^2k}{(2\pi)^2} V_{eff}(\mathbf{k}-\mathbf{k}')\Delta(\mathbf{k}')\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}$

$$\epsilon(k) = \sqrt{(\hbar^2k^2/2m - \mu)^2 + |\Delta(k)|^2}; \quad \mu \approx E_F$$

$$T_c \approx E_F \exp(-3\pi/4k_F r_*)$$

$\Delta(\mathbf{k}) = \Delta \exp(i\phi_k) \quad p_x + ip_y$ state ($l = \pm 1$)

Superfluid transition

In fact, Kosterlitz-Thouless transition (2D!)

However, for weak interactions T_{KT} is very close to T_{BCS}

Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$
This is the case for the atoms

Polar molecules

Anomalous scattering in $1/r^3$ potential $\rightarrow$ Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m}(kr_*); \qquad |k| = |k'|$$

$$T_c \propto \exp\left(-\frac{1}{\nu(k_F)|V_{eff}(k_F)|}\right); \qquad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto \exp\left(-\frac{3\pi}{4k_F r_*}\right)$$

Transition temperature

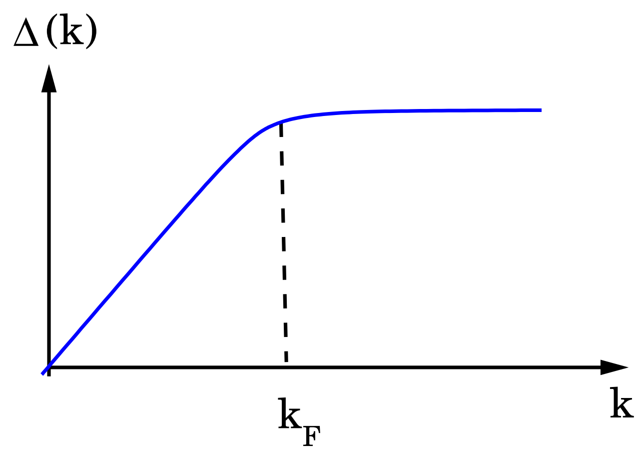
Do better than simple BCS. Reveal the role of short-range physics

Renormalized gap equation

$$\Delta(\mathbf{k}') = -\int f(\mathbf{k}', \mathbf{k})\Delta(\mathbf{k}) \left\{ \frac{\tanh[\epsilon(k)/2T]}{2\epsilon(k)} - \frac{1}{(E_k - E_{k'} - i0)} \right\} \frac{d^2k}{(2\pi)^2}$$

$\Delta(\mathbf{k}) = \Delta(k)\exp(i\phi_k)$; $f(\mathbf{k}', \mathbf{k}) = f(k', k)\exp[i(\phi_k - \phi_{k'})]$ scattering amplitude

enterline$\Delta(k) = \Delta(k_F) f(k, k_F)/f(k_F, k_F)$



Exact low-energy solution $f = -(8\pi/3)d^2k + (\pi/2)d^2 r_* k^2 \ln k$

Related results for the off-shell scattering amplitude

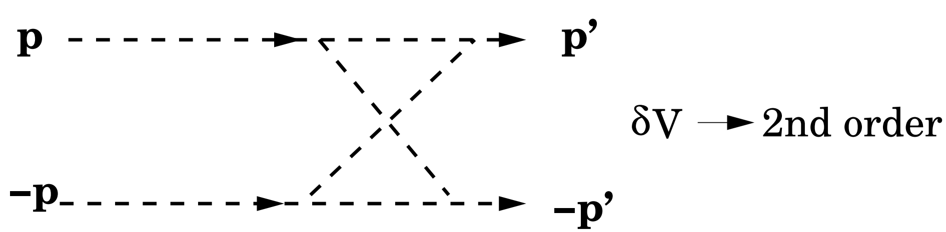
Manipulate T_c?

Include k^2-term $f = -\left(\frac{8\pi}{3}\right) d^2 k + (\frac{1}{2}\pi d^2 r_* k^2 \ln[k r_* u]$

$u \to$ short-range physics

$$T_c = \frac{2e^C}{\pi} E_F \exp\left\{-\frac{3\pi}{4k_F r_*} - \frac{9\pi^2}{64}\ln[k_F r_* u]\right\}$$

Take into account second-order Gor'kov-Melik-Barkhudarov processes



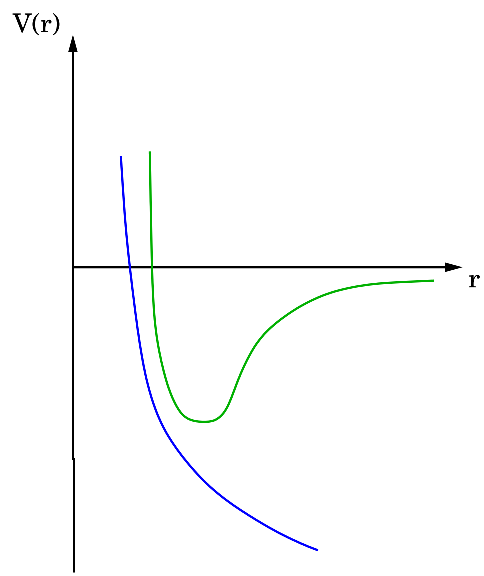
$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp\left\{-\frac{3\pi}{4k_F r_*}\right\}; \quad E_* = \frac{\hbar^2}{2mr_*^2} \gg E_F$$

κ depends on short-range physics and can be varied within 2 orders of magnitude

Collisional stability and T_c

p-wave atomic superfluids: $\mathrm{BCS} \Rightarrow T_c \to 0$ Resonance $\Rightarrow$ collisional instability

Polar molecules $\Rightarrow$ sufficiently large T_c and collisional stability



$\alpha_{in} = A\frac{\hbar}{m}(kr_*)^2;$ $A \Rightarrow 10^{-3} - 10^{-4}$ $\alpha_{in} \to (10^{-8} - 10^{-9})$ cm^2/s

LiK molecules $\to$ $d \simeq 3.5$ D $r_* \approx 4000a_0$

$n = 2 \times 10^8$ cm$^{-2} \Rightarrow$ $E_F = 2\pi\hbar^2 n/m = 120$ nK $T_c \approx 10$ nK; $\tau \sim 2$s