Dipolar quantum gases

Gora Shlyapnikov
LPTMS, Orsay, France
University of Amsterdam

Outline

- Experiments with ultracold dipolar particles
- Scattering problem. Dipolar bosons
- Dipolar BEC. Stability problem
- Dipolar Fermi gas. Scattering problem
- Superfluid pairing
- Several ideas

Hannover, April 11, 2014
Novel object - Dipolar gas

Polar molecules or atoms with a large magnetic moment

Dipole-dipole interaction

\[ V_d = \frac{d_1 d_2 R^2 - 3 (\vec{d}_1 \vec{R}) (\vec{d}_2 \vec{R})}{R^5} \sim \frac{1}{R^3} \]

long-range, anisotropic

repulsion

attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules \( d \) from 0.6 \( D \) for KRb to 5.5 \( D \) for LiCs
Atoms with large $\mu$

Remarkable experiments with Cr atoms ($\mu = 6\mu_B \Rightarrow d \approx 0.05$ D)

T. Pfau group (Stuttgart)

Effects of the dipole-dipole interaction in the dynamics
Stability diagram of trapped dipolar BEC

Spinor physics in chromium experiments at Villetaneuse, B. Laburthe-Tolra

Now disprosium ($\mu = 10\mu_B$, (B. Lev))
and erbium ($\mu = 7\mu_B$, (F. Ferlaino)) are in the game
Polar molecules. Creation of ultracold clouds

- Buffer gas cooling (Harvard, J. Doyle)
  \[ n \sim 10^8 \text{cm}^{-3} \quad T \approx 300\text{mK} \]

- Stark deceleration (Meijer, Berlin; JILA)
  \( \text{H}_3, \text{ND}_3, \text{CO}, \text{etc.} \); \( T \sim 1\text{mK} \) and low density

- Optical collisions (photoassociation, D. DMille group, JILA, elsewhere)
Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb fermionic molecules to the ground rovibrational state

JILA, D. Jin, J. Ye groups

\[ n \sim 10^{12} - 10^{13} \text{ cm}^{-3} \]

\[ T \approx 200nK \sim E_F \]

Ground-state LiCs molecules at Heidelberg

Ground-state RbCs molecules in Innsbruck

Ground-state KRb bosonic molecules in Tokyo

Experiments with NaK (MIT, Munich, Hannover, Trento) and KCs (Innsbruck) molecules
Ultracold chemistry

Ultracold chemical reactions: $\text{KRb} + \text{KRb} \rightarrow \text{K}_2 + \text{Rb}_2$

New trends in ultracold chemistry

Suppress instability $\rightarrow$ induce intermolecular repulsion

For example, 2D geometry with dipoles perpendicular to the plane

Reduction of the decay rate by 2 orders of magnitude at JILA

Select non-reactive molecules, like NaK, KCs, RbCs
Theoretical studies

- Innsbruck group (P. Zoller, G. Pupillo, M.A. Baranov et al). Large variety of proposals including bilayer systems, Rydberg atoms etc.
- Trento group (S. Stringari et al). Excitation modes etc
- Harvard group (E. Demler, M. Lukin et al). Multilayer systems etc
- Hannover group (L. Santos et al). Spinor and dipolar systems
- Tokyo group (M. Ueda et al) Spinor and dipolar systems
- Cambridge group (N.R. Cooper, Jesper Levinsen). Novel states
- Rice group (H. Pu et al). Excitations and stability etc
- Maryland group (S. Das Sarma et al) Fermi liquid behavior etc
- Taipei group (D.-W. Wang et al)
- Barcelona group (M. Lewenstein et al)
Dipole-diple scattering

\[ V_d = \frac{d^2}{r^3} \left( 1 - 3 \cos^2 \theta_{rd} \right) \sim Y_{20}(\theta_{rd}) \]

Wave function of the relative motion

\[ \psi_{in} \rightarrow \sum_{l,m} \psi_{kl}(r) i^l Y_{lm}^*(\theta_{kd}, \varphi_{kd}) Y_{lm}(\theta_{rd}, \varphi_{rd}) \]
\[ \psi_{out} \rightarrow \sum_{l',m'} \psi_{k'l'}(r) i^{l'} Y_{l'm'}^*(\theta_{kd}, \varphi_{kd}) Y_{l'm'}(\theta_{rd}, \varphi_{rd}) \]

Scattering matrix \( \sim \int Y_{lm}^* Y_{l'm'} Y_{20} d\Omega_{rd} \)

\( V_d \) couples all even \( l \), and all odd \( l \), but even and odd \( l \) are decoupled from each other
Radius of the dipole-dipole interaction

\[
\left( -\frac{\hbar^2}{m} \Delta + V_d(\vec{r}) \right) \psi(\vec{r}) = \frac{\hbar^2 k^2}{m} \psi(\vec{r})
\]

\[
\frac{\hbar^2}{mr^2} = \frac{d^2}{r^3} \Rightarrow r^*_* \approx \frac{md^2}{\hbar^2}
\]

\[r \gg r_* \rightarrow \text{free relative motion}\]

\[r_* \approx 10^6 \div 10^3 a_0 \quad \text{polar molecules}\]

\[r_* \approx 50a_0 \rightarrow \text{chromium atoms}\]

\[kr_* \ll 1 \rightarrow \text{Ultracold limit}\]

\[T \ll 1 mK \quad \text{for Cr}\]
Scattering amplitude I

\[ V(\vec{r}) = U(\vec{r}) + V_d(\vec{r}) \]

\[ f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i \vec{k}_f \cdot \vec{r}} d^3 r \]

Ultracold limit \( k r_\ast \ll 1 \)

\[ V_d = 0 \Rightarrow f = g = \frac{4 \pi \hbar^2}{m} a \]

What \( V_d \) does?

\[ k = 0 \rightarrow g = \int \psi^*_0(\vec{r})(U(\vec{r}) + V_d(\vec{r}))d^3 r = \text{const}; \quad r \lesssim r_\ast \]

\( g \) may depend on \( d \) and comes from all even \( l \)
Scattering amplitude II

\[ k \neq 0 \]

\[ f = \int \psi_{k_i}^* (\vec{r}) V (\vec{r}) e^{i \vec{k}_f \vec{r}} d^3 r \]

\[ r \lesssim r_* \rightarrow \text{put } k = 0 \rightarrow g \]

\[ r \gg r_* \rightarrow \psi_{k_i} = e^{i \vec{k}_i \vec{r}} \]

\[ f = \int V_d (\vec{r}) e^{i \vec{q} \vec{r}} d^3 r \rightarrow \frac{4\pi d^2}{3} (3 \cos^2 \theta_{qd} - 1); \quad \vec{q} = \vec{k}_f - \vec{k}_i \]

\[ f = g + \frac{4\pi d^2}{3} (3 \cos^2 \theta_{qd} - 1) \]
Dipolar BEC I

Uniform gas \( g > \frac{4\pi d^2}{3} \rightarrow f > 0 \Rightarrow \text{stable BEC} \)

\[
H = \int d^3 \left[\psi^\dagger(\vec{r}) \left( -\frac{\hbar^2}{2m} \Delta \right) \psi(\vec{r}) + \frac{1}{2} g \psi^\dagger(\vec{r}) \psi^\dagger(\vec{r}) \psi(\vec{r}) \psi(\vec{r}) + \frac{1}{2} \int d^3 r' \psi^\dagger(\vec{r}) \psi^\dagger(\vec{r}') V_d(\vec{r} - \vec{r}') \psi(\vec{r}') \psi(\vec{r}) \right]
\]

Bogoliubov approach \( \psi = \psi_0 + \delta \Psi \rightarrow \text{bilinearar Hamiltonian} \)

\[
H_B = \frac{N^2}{2V} g + \sum_k \left[ \frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + n \left( g + \frac{4\pi d^2}{3} \left( 3 \cos^2 \theta_k - 1 \right) \right) a_k^\dagger a_k \right. \\
\left. + \frac{n}{2} \left( g + \frac{4\pi d^2}{3} \left( 3 \cos^2 \theta_k - 1 \right) \right) \left( a_k^\dagger a_{-k} + a_k a_{-k} \right) \right]
\]
Excitation spectrum

\[ \epsilon_k = \sqrt{E_k^2 + 2E_k n \left( g + \frac{4\pi d^2}{3} \left( 3 \cos^2 \theta_k - 1 \right) \right)} \]

\[ g > \frac{4\pi d^2}{3} \rightarrow \text{dynamically stable BEC} \]

\[ g < \frac{4\pi d^2}{3} \rightarrow \text{complex frequencies at small } k \]

and \[ \cos^2 \theta_k < \frac{1}{3} \rightarrow \text{collapse} \]
Trapped dipolar BEC

Cylindrical trap

\[ V_h = \frac{m}{2} (\omega^2 \rho^2 + \omega^2 z^2) \]

Gross-Pitaevskii equation

\[
\left[ -\frac{\hbar^2}{2m} \Delta + V_h(\vec{r}) + g\psi^2_0 + \int \psi_0(\vec{r}')^2 V_d(\vec{r} - \vec{r}') d^3 r' \right] \psi_0(\vec{r}) = \mu \psi_0(\vec{r})
\]

Important quantity

\[ V_{eff} = g \int \psi_0^4(\vec{r}) d^3 r + \int \psi_0^2(\vec{r}') V_d(\vec{r} - \vec{r}') \psi_0^2(\vec{r}) d^3 r d^3 r' \]

\[ V_{eff} > 0 \text{ or } V_{eff} < 0 \text{ and } |V| < \hbar \omega \]

\[ g = 0 \rightarrow N < N_c \rightarrow \text{suppressed low } k \text{ instability} \]

(Santos et.al, 2000)
It is sufficient?

\[
\frac{\omega_z}{\omega_\rho} = 1
\]

\[
\frac{\omega_z}{\omega_\rho} > 6.25
\]

\[
V_{\text{eff}} < 0
\]

\[
V_{\text{eff}} > 0
\]
Stability problem

\[
\langle V_d \rangle = \int n_0(\vec{r}') V_d(\vec{r}' - \vec{r}) d^3r = -d^2 \frac{\partial^2}{\partial z^2} \int \frac{n_0(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r
\]

\[
V_d = -d^2 \frac{\partial^2}{\partial z^2} \frac{1}{|\vec{r} - \vec{r}'|} - \frac{4\pi d^2}{3} \delta(\vec{r} - \vec{r}')
\]

Large \(N\) \(\Rightarrow\) Thomas-Fermi BEC

\[
n_0 = n_{0 \text{ max}} \left(1 - \frac{z^2}{R_z^2} - \frac{\rho^2}{R_\rho^2}\right)
\]

Eberlein et. al (2005)

\[
g > \frac{4\pi d^2}{3} \rightarrow \text{stable at any } N
\]
Example

\[ \frac{\omega_z}{\omega_\rho} = 2 \]

\[ \frac{\omega_z}{\omega_\rho} = 1 \]

\[ \frac{\omega_z}{\omega_\rho} = 0.2 \]

\[ \varepsilon = \frac{4\pi d^2}{3g} \]
Experiment with Cr

\[ g > \frac{4\pi d^2}{3} \]

(\(\mu = 6\mu_B!\))

(T. Pfau, Stuttgart) BEC \((n \sim 10^{14} cm^{-3})\)

effect of the dipole-dipole interaction (small)
Pancake dipolar BEC

\[ g < g_d = \frac{4\pi d^2}{3} \]  

Thomas-Fermi in the \( z \) direction

Extreme pancake (\( \omega_\rho = 0 \))

\[ l_0 = \left( \frac{\hbar}{m\omega_z} \right)^{1/2} \]

\( V_d + g \) (short-range)  \( g > 0 \)

\[ g/g_d = 1.06 \quad \mu/\hbar\omega_z = 46 \]

\[ g/g_d = 0.94 \quad \mu/\hbar\omega_z = 53 \]

Roton structure \( \Rightarrow \) decrease of the interaction amplitude
Roton-minaxon structure

Roton minimum can be stable at zero
Instability!

\[
\frac{g_c}{g_d}
\]

\[
\frac{\mu}{\hbar \omega}
\]

(L. Santos et al., 2003)
Stuttgart experiment

Trap aspect ratio $\lambda = \omega_z / \omega_r$

$a_{crit} / a_0$

$E(\sigma_r, \sigma_z) / N\hbar\omega$

$p. 21$
Ideas from superfluid $^4\text{He}$

Why the roton-maxon structure? R. Feynman

$$\epsilon_q = \frac{q^2}{2mS(q, \epsilon)}$$

How to put the roton minimum higher (*) or lower (**)?

What happens? (S. Balibar, P. Nozieres, L. Pitaevskii)

(*) negative pressure (acoustic pulses)  
(**) increase the pressure

Metastable liquid

(**) $\Rightarrow$ supersolid (density wave), loss of superfluidity, or?
Prehistory

L.P. Pitaevskii (1981)

$^4$He

$v > v_c$

$v > v_c \implies$ Density wave
Quasi2D dipolar BEC at $T = 0$

\[ l_0 = \left( \frac{\hbar}{m\omega_z} \right)^{1/2} \]

\[ \varphi_0(z) = \frac{1}{\pi^{1/4}l_0^{1/2}} \exp \left\{ -\frac{z^2}{2l_0^2} \right\} \]

short range interaction $(g)$ + dipole-dipole

Consider $0 < g \ll \frac{4\pi d^2}{3}$. Then, for $qr_* \ll 1$

\[ V_{\vec{q}\vec{p}} = g(1 - C|\vec{q} - \vec{p}|) \]

where

\[ C = \frac{2\pi d^2}{g} \]
\[ \hat{H} = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a^{\dagger}_{\vec{k}} a_{\vec{k}} + \frac{g}{2} \sum_{\vec{k}, \vec{q}, \vec{p}} (1 - C |\vec{q} - \vec{p}|) a^{\dagger}_{\vec{k} + \vec{q}} a^{\dagger}_{\vec{k} - \vec{q}} a_{\vec{k} + \vec{p}} a_{\vec{k} - \vec{p}} \]

\[ \varepsilon_k^2 = E_k^2 + 2\mu E_k (1 - C k) \]

\[ k_r = \frac{3}{2} \left( 1 + \sqrt{\frac{C^2}{\xi^4} - \frac{8}{9\xi^2}} \right) \]

\[ \xi = \frac{\hbar}{m n g}. \]
Rotonization

$$\xi \geq C \geq \frac{\sqrt{8}}{3} \xi$$

The roton minimum touches zero for

$$C = \xi \Rightarrow k_r = \frac{2C}{\xi}$$

For $C > \xi$ we have collapse. No stable supersolid state

Pedri/Shlyapnikov; Cooper/Komiteas (2007)
What does the dipole-dipole interaction do in a Fermi gas?

- Single component gas

The dipole-dipole scattering amplitude is independent of $|k|$ at any orbital angular momenta allowed by the selection rules.

$\text{Long-range contribution } \sim d^2$

$\text{Short-range contribution } \sim \kappa^2$

\[\text{omit}\]

Universal result for $f$
Physical picture. Stability

\[ r \lesssim r_* \]

\[ n d^2 \text{ significantly larger than } E_F = \frac{(6\pi^2 n)^{2/3}}{2m}, \]

which is \( k_F r_* \) significantly larger than 1, leads to collapse
Odd-$l$ scattering amplitude

\[ f = \frac{1}{2} \int \left( e^{i\vec{k}_i \vec{r}} - e^{-i\vec{k}_i \vec{r}} \right) V_d(\vec{r}) \left( e^{-i\vec{k}_f \vec{r}} - e^{i\vec{k}_f \vec{r}} \right) d^3r = 4\pi d^2 (\cos^2 \theta_{dq_-} - \cos^2 \theta_{dq_+}) \]

\[ \vec{q}_\pm = \vec{k}_i - \vec{k}_f \]

\[ e^{i\vec{k}\vec{r}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} j_l(kr) i^l Y_{lm}(\hat{r}) Y_{lm}^*(\hat{k}) \]

Partial amplitude \[ f(l_i m_i; l_f m_f) \]

\[ f(10; 10) = -\frac{6d^2}{5} \cos \theta_{d_ki} \cos \theta_{d_kf} \]
**Superfluid $p$-wave pairing**

\[ nd^2 \ll E_F \]

Analog of \( a \to \sim \frac{md^2}{\hbar^2} (r_*) \)

\[ \Delta = g\langle \psi_\uparrow \psi_\downarrow \rangle \sim E_F \exp \left( -\frac{1}{\chi} \right); \lambda \sim \frac{1}{k_F r_*}; \]

\[ \Delta \propto E_F \exp \left\{ -\frac{\pi E_F}{12nd^2} \right\} \]

Cooper pairs are superpositions of all odd angular momenta (for \( m_l = 0 \))
Transition temperature

\[ \frac{nd^2}{E_F} \ll 1 \]

\[ T_c = 1.44E_F \exp \left\{ -\frac{\pi E_F}{12nd^2} \right\} \]

Baranov et.al (2002)
GM correction included

\[ \Delta \rightarrow \text{anisotropic} \propto \sin \left( \frac{\pi}{2} \cos \theta \right) \]

- Maximum in the direction of the dipoles.
- Vanishes in the direction perpendicular to the dipoles

- p. 31
Distinguished features

Anisotropic gap $\rightarrow$ gapless excitations in the direction perpendicular to the dipoles. Damping rates etc.

Anisotropy $\rightarrow$ different from $p$-wave superfluid B and A phases of $^3$He. In B $\Delta$ is isotropic, and in A it vanishes only at 2 points on the Fermi sphere ($\theta = 0$ and $\theta = \pi$)
Value of $T_c$

\[ d \sim 0.1 \div 1 \mathcal{D} \]

NaK $\Rightarrow$ $d = 2.7\mathcal{D}$ and can be made $0.5\mathcal{D}$ ($r_\ast = 2500\ \text{Å}$) in a certain electric field

\[ \frac{T_c}{E_F} \rightarrow 0.025 \text{ at } n \rightarrow 6 \times 10^{12}\text{cm}^{-3} \ (E_F \approx 400\ \text{nK}) \]

$(T_c \rightarrow 10nK)$

One easily achieves the strongly interacting regime
2D dipolar Fermi gas

\[ l_0 = \left( \frac{\hbar}{m \omega_z} \right)^{1/2} \]

\[ \varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp \left\{ \frac{-z^2}{2l_0^2} \right\} \]

2-component Fermi gas (\(\uparrow\) and \(\downarrow\)). Short range \(g > 0\)

\[ H = \sum_k \frac{\hbar^2 k^2}{2m} \left( a_{k\downarrow}^\dagger a_{k\downarrow} + a_{k\uparrow}^\dagger a_{k\uparrow} \right) + \]
\[ + |g| \sum_{k,p,q} (1 - C|\vec{q} - \vec{p}|) a_{k+q\uparrow}^\dagger a_{k-q\downarrow}^\dagger a_{k-p\downarrow} a_{k+p\uparrow} \]

\[ C = \frac{2\pi d^2}{|g|} \]
Interesting problem

$s$-wave interaction on the Fermi surface

\[ |g|(1 - 4Ck_F/\pi) \]

\[ k_F C > \frac{\pi}{4} \rightarrow 8d^2k_F > |g| \rightarrow \text{superfluidity} \]

\[ k_F C < \frac{\pi}{4} \rightarrow 8d^2k_F < |g| \rightarrow \text{no superfluidity} \]

Quantum transition to a normal state with decreasing density

Superfluid pairing for tilted dipoles $\rightarrow$ Baranov/Sieberer (2011)
1D dipolar Fermi gas

\[
\varphi_0(\rho) = \frac{1}{\pi^{1/2} l_0} \exp \left\{ \frac{-\rho^2}{2l_0^2} \right\}
\]

short range \quad g > 0

\[
H = \sum_k \frac{\hbar^2 k^2}{2m} \left( a_{k\downarrow}^\dagger a_{k\downarrow} + a_{k\uparrow}^\dagger a_{k\uparrow} \right) + \\
+ |g| \sum_{k,p,q} \left( 1 + B |\vec{q} - \vec{p}|^2 \ln(|\vec{q} - \vec{p}| l_0) \right) a_{k+q\uparrow}^\dagger a_{k-q\downarrow}^\dagger a_{k+p\downarrow} a_{k-p\uparrow}
\]

\[
B = \frac{d^2}{|g|}
\]
Quantum transition

Interaction at the Fermi points

\[ g_{eff} = |g| \left[ 1 + 2B(k_F l_0)^2 \ln(k_F l_0) \right] \]

\[ g_{eff} < 0 \rightarrow \text{superfluid} \]

\[ g_{eff} > 0 \rightarrow \text{ordinary Luttinger liquid} \]

Quantum transition superfluid-Luttinger liquid with decreasing density
Are there novel many-body states?

Does the dipole-dipole interaction lead to the emergence of novel phases for identical fermions?
Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)

The number of zero-energy states exponentially grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics $\Rightarrow$ Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

Topologically protected state for quantum information processing
$p$-wave resonance for fermionic atoms

$p$-wave resonance  Experiments at JILA, ENS, Melbourne, Tokyo, elsewhere

$$BCS \Rightarrow T_c \sim \exp \left( \frac{1}{(k_F b)^2} \right) \text{ practically zero}$$

Molecular and strongly interacting regimes $\Rightarrow$ rather high $T_c$, but collisional instability

Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio

\[ \text{p–wave molecules} \quad \text{unstable} \]

\[ \text{unstable} \quad \text{SIR} \quad \text{BCS stable} \]

\[ g \quad B \]

\[ \text{BCS stable} \]
RF-dressed polar molecules in 2D. Innsbruck idea

Dressed states

\[ |+\rangle = \alpha |0, 0\rangle + \beta |1, 1\rangle; \quad , |\rangle = \beta |0, 0\rangle - \alpha |1, 1\rangle \]

\[ \alpha = -\frac{A}{\sqrt{A^2 + \Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2 + \Omega^2}}; \quad A = \frac{1}{2} (\delta + \sqrt{\delta^2 + 4\Omega^2}) \]

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency

\[ d_c = 2^{1/2} \alpha \beta d \]

Large \( r \rightarrow \) \( V_{eff} = \langle (1 - 3 \cos^2 \phi) \rangle \frac{d_c^2}{r^3} = - \frac{d_c^2}{2r^3}; \quad r_* = m d_c^2 / 2\hbar^2 \]
Fermionic RFD molecules. Superfluid transition

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the $p$-wave scattering ($l = \pm 1$)

\[
\hat{H} = \int d^2r \, \hat{\Psi}^\dagger(r) \{-\left(\frac{\hbar^2}{2m}\right)\Delta + \int d^2r' \hat{\Psi}^\dagger(r') V_{eff}(r - r') \hat{\Psi}(r') - \mu\} \hat{\Psi}(r)
\]

\[
\Delta(r - r') = \langle V_{eff}(r - r') \hat{\Psi}(r) \hat{\Psi}(r') \rangle
\]

Gap equation

\[
\Delta(k) = -\int \frac{d^2k}{(2\pi)^2} V_{eff}(k - k') \Delta(k') \frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}
\]

\[
\epsilon(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + |\Delta(k)|^2}; \quad \mu \approx E_F
\]

\[
T_c \approx E_F \exp\left(-\frac{3\pi}{4k_F r_\ast}\right)
\]

\[
\Delta(k) = \Delta \exp(i\phi_k) \quad \text{$p_x + ip_y$ state ($l = \pm 1$)}
\]
Superfluid transition

In fact, Kosterlitz-Thouless transition (2D!)
However, for weak interactions $T_{KT}$ is very close to $T_{BCS}$

Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$
This is the case for the atoms

Polar molecules

Anomalous scattering in $1/r^3$ potential → Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m} (kr_*); \quad |k| = |k'|$$

$$T_c \propto \exp \left( -\frac{1}{\nu(k_F)|V_{eff}(k_F)|} \right); \quad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto \exp \left( -\frac{3\pi}{4k_F r_*} \right)$$
**Transition temperature**

Do better than simple BCS. Reveal the role of short-range physics

Renormalized gap equation

\[
\Delta(k') = - \int f(k', k) \Delta(k) \left\{ \frac{\tanh[\epsilon(k)/2T]}{2\epsilon(k)} - \frac{1}{(E_k - E_{k'} - i0)} \right\} \frac{d^2k}{(2\pi)^2}
\]

\[\Delta(k) = \Delta(k) \exp(i\phi_k); \quad f(k', k) = f(k', k) \exp[i(\phi_k - \phi_{k'})]\]

scattering amplitude

\[\Delta(k) = \Delta(k_F) f(k, k_F) / f(k_F, k_F)\]

Exact low-energy solution

\[f = -(8\pi/3)d^2k + (\pi/2)d^2r_0k^2 \ln k\]

Related results for the off-shell scattering amplitude
Manipulate $T_c$?

Include $k^2$-term $f = -\left(\frac{8\pi}{3}\right)d^2k + \left(\frac{1}{2}\pi d^2r_*k^2\ln[kr_*u]\right)$

$u \rightarrow$ short-range physics

$$T_c = \frac{2e^C}{\pi} E_F \exp\left\{-\frac{3\pi}{4k_Fr_*} - \frac{9\pi^2}{64}\ln[k_Fr_*u]\right\}$$

Take into account second-order Gor’kov-Melik-Barkhudarov processes

$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp\left\{-\frac{3\pi}{4k_Fr_*}\right\}; \quad E_* = \frac{\hbar^2}{2mr_*^2} \gg E_F$$

$k$ depends on short-range physics and can be varied within 2 orders of magnitude
Collisional stability and $T_c$

$p$-wave atomic superfluids: $\text{BCS} \Rightarrow T_c \rightarrow 0$ Resonance $\Rightarrow$ collisional instability

Polar molecules $\Rightarrow$ sufficiently large $T_c$ and collisional stability

\[
\alpha_{in} = A \frac{\hbar}{m} (kr_*)^2; \quad A \Rightarrow 10^{-3} - 10^{-4} \quad \alpha_{in} \rightarrow (10^{-8} - 10^{-9}) \text{ cm}^2/\text{s}
\]

LiK molecules $\Rightarrow d \approx 3.5 \text{ D} \quad r_* \approx 4000a_0$

\[n = 2 \times 10^8 \text{ cm}^{-2} \Rightarrow E_F = 2\pi \hbar^2 n/m = 120 \text{ nK} \quad T_c \approx 10 \text{ nK}; \quad \tau \sim 2\text{s}\]