Rapidly rotating quantum gases.

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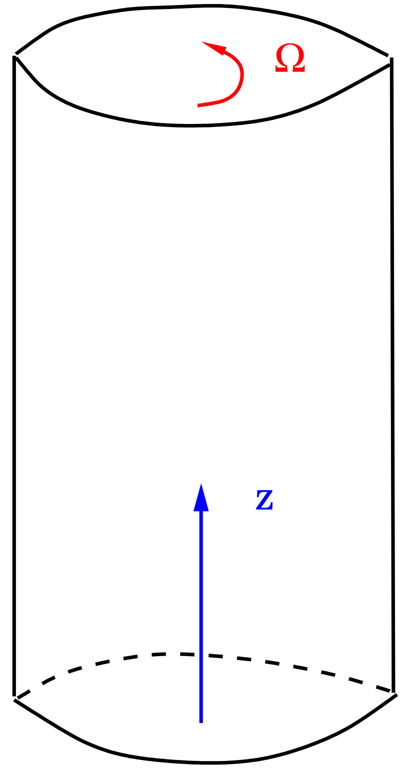
Outline

- Introduction. Vortices in supetrfluids
- Rapid rotation. BEC in the lowest Landau level.
- Quantum transitions. iVortex lattice of trapped BEC
- Excitation modes of the vortex lattice in the LLL
- One-body density matrix
- Melting of the lattice. Strongly correlated states

Hannover, April 24, 2014

Vortices in superfluids

Rotating Bose superfluid at $T = 0$. No motion



$E_{\rm rot} = E - L_z\Omega$ is minimum. Superfluid motion around microscopically narrow lines at which the condition $\mathrm{curl}\,\mathbf{v}_s = 0$ is violated

Circulation of the superfluid velocity around a vortex line

$$\oint \mathbf{v}_s d\mathbf{l} = \oint \frac{\hbar}{m}\nabla S d\mathbf{l} = \frac{2\pi\hbar}{m}s; \quad S = \frac{1}{\hbar}[m\mathbf{v}_s\mathbf{r} - (mv_s^2/2 + \mu)t]$$

Vortices in superfluids

In the reference frame where the liquid moves with velocity $\mathbf{v}_s$:

$$\Psi_0 = \sqrt{n_0}\exp(iS) \Rightarrow \quad \mathbf{v}_s = \frac{\hbar}{m}\nabla S$$

Single valued $\Psi_0] \Rightarrow$ integer s. The circulation is quantized in units of $\hbar/m$.

Vortex line along z. Streamlines of $\mathbf{v}_z$ are circles perpendicular to z:

$$v_s = s\frac{\hbar}{mr}$$

Angular momentum $L_z = \int n_s m v_s r d^3 r = \pi s \mathcal{R}^2 \mathcal{L} \hbar n_s$

Energy associated with the vortex $E_v = \frac{1}{2}\int n_s m v_s^2 d^3 r = \pi n_s m s^2 \left(\frac{\hbar}{m}\right)^2 \ln\left(\frac{\mathcal{R}}{r_c}\right)$

Critical rotation frequency $\Rightarrow E_{\mathrm{rot}} = E_v - \Omega_c L_z = 0 \Rightarrow$

$$\Omega_c = \frac{E_v}{L_z} = \frac{\hbar}{m\mathcal{R}^2}\ln\left(\frac{\mathcal{R}}{r_c}\right)$$

This is for $|s| = 1$. The states with the charge $|s| > 1$ are unstable

Vortices in superfluids

For Ω greatly exceeding $\Omega_c \Rightarrow$ many vortices

$$\oint \mathbf{v}_s d\mathbf{l} = 2\pi N_v \frac{\hbar}{m}$$

$N_v \gg 1 \Rightarrow$ relations for a rotating rigid body

$v_s = \Omega r$ and $|\mathrm{curl}\mathbf{v}_s| = 2\Omega$, so that $\oint \mathbf{v}_s d\mathbf{l} = 2\Omega A$

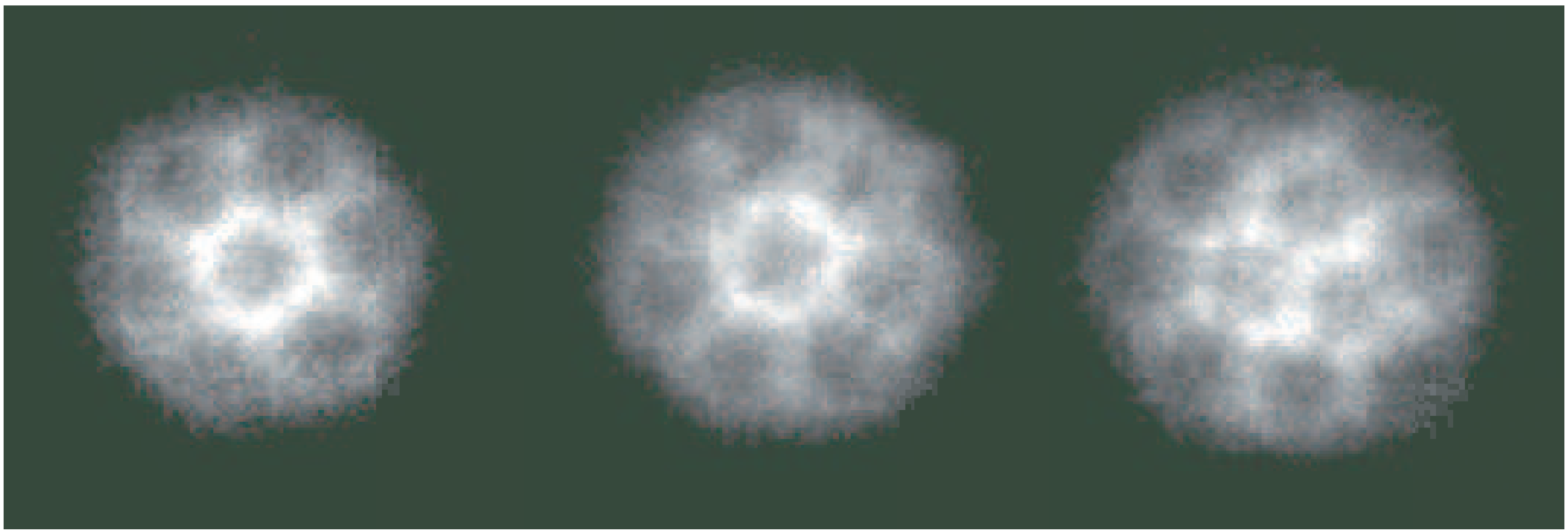
$$n_v = \frac{N_v}{A} = \frac{m\Omega}{\pi\hbar}$$

Experiments

Superfluid ^{4}He W.F. Vinen (1961), R.E. Packard/T.M. Sanders (1972)

Bose-condensed ultracold atomic gases

JILA (E.A. Cornell), ENS (J. Dalibard), MIT (W. Ketterle)



Gross-Pitaevskii equation for the vortex state

Straight vortex line along the z-axis and $|s| = 1$ L.P. Pitaevskii (1963)

$$\psi_0 = \sqrt{n_0} f(r) \exp(i\phi) \quad r = \sqrt{x^2 + y^2}$$

Laplacian $\Rightarrow \quad \Delta_{\mathbf{r}} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$

Gross-Pitaevskii equation $-\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right) f + n_0 g |f|^2 f - \mu f = 0$

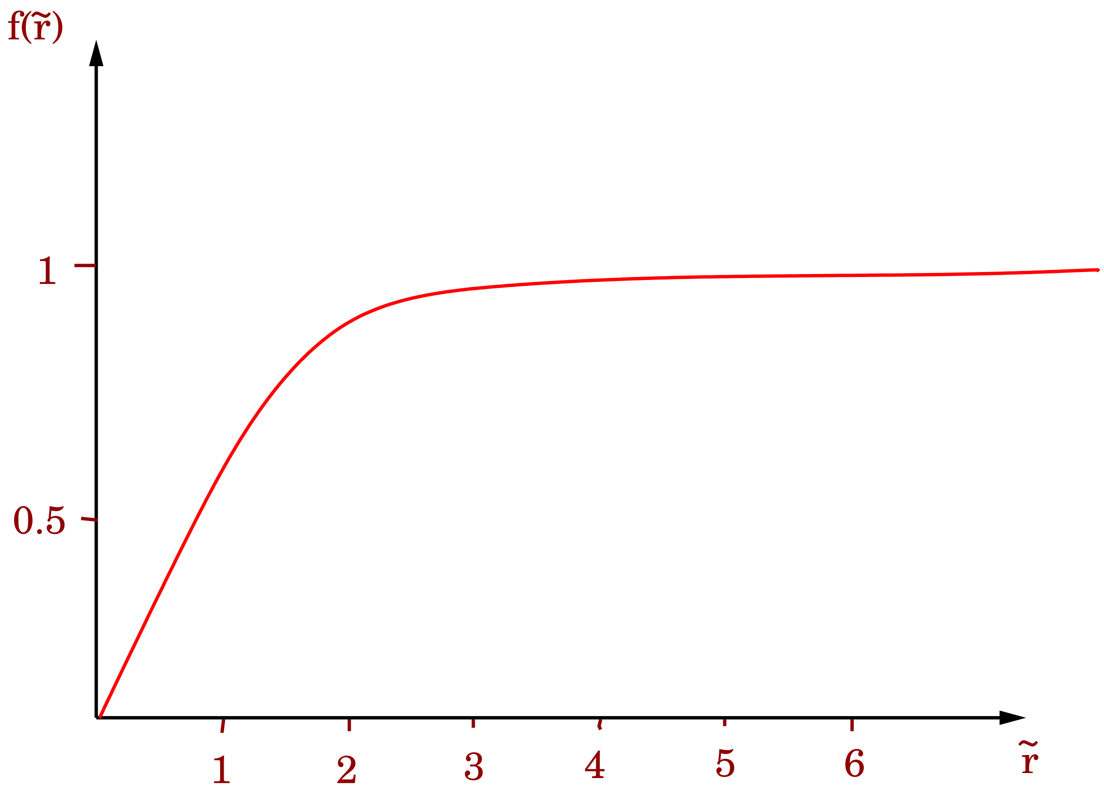
Energy scale $\mu = n_0 g$ and length scale $\xi = \hbar/\sqrt{m\mu}$

$$\frac{d^2 f}{d\tilde{r}^2} + \frac{1}{\tilde{r}}\frac{df}{d\tilde{r}} - \frac{f}{\tilde{r}^2} - f^3 + f = 0; \quad \tilde{r} = \sqrt{2} r/\xi$$

$$f \propto \tilde{r}, \qquad \tilde{r} \to 0$$

$$f \propto \left(1 - \frac{1}{2\tilde{r}^2}\right), \qquad \tilde{r} \to \infty$$

Gross-Pitaevskii equation for the vortex state

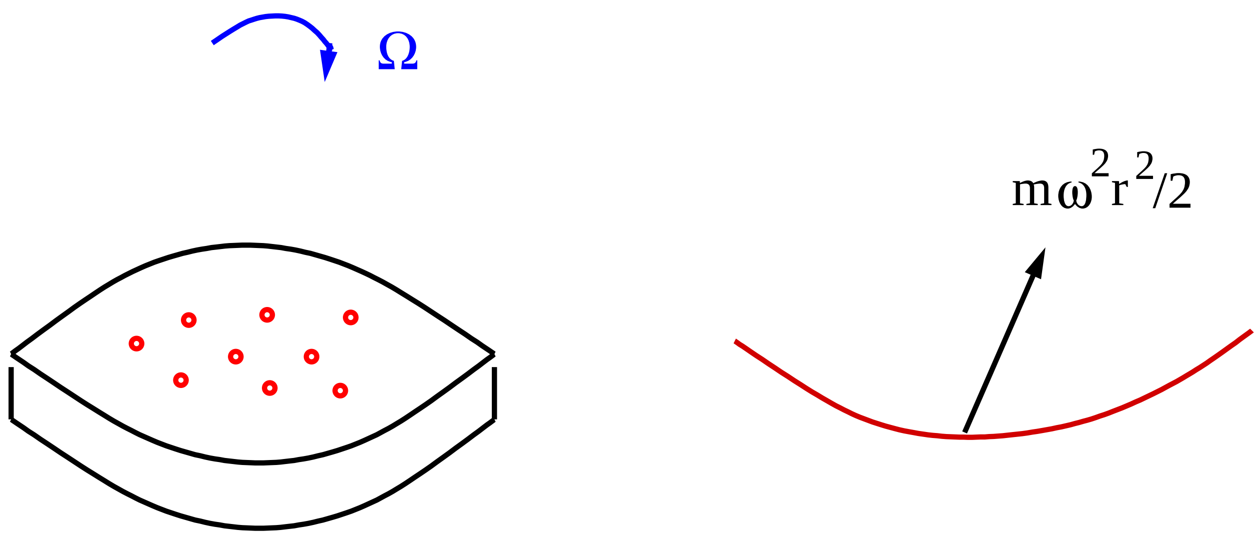


$r_c \sim \xi$ and using $\mu = n_0 g$ and $g = 4\pi\hbar^2 a/m \Rightarrow \ \ r_c \sim 1/\sqrt{n_0 a} \gg n_0^{-1/3}$ ($n_0 a^3 \ll 1$)

Topological quantum number, circulation. The vortex can only decay when going to the border of the system

Rapid rotation. Single particle problem

Non-interacting particles in the x, y plane rotating with frequency Ω around the z-axis in the harmonic potential $V(r) = m\omega^2 r^2/2$; $\; r = \sqrt{x^2 + y^2}$



Single particle Hamiltonian $\hat{H}^{(1)} = -\frac{\hbar^2}{2m}\Delta_{\mathbf{r}} + \frac{m\omega^2 r^2}{2} - \hbar\Omega\hat{L}_z$

$$\hat{L}_z = i\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right); \quad \Omega < \omega$$

$$\hat{H}^{(1)} = -\frac{\hbar^2}{2m}(\nabla_{\mathbf{r}} - i\mathbf{A})^2 + (\omega^2 - \Omega^2)\frac{mr^2}{2}$$

$$\mathbf{A} = m\Omega(\hat{\mathbf{z}} \times \mathbf{r})/\hbar$$

Landau levels

Common eigenbasis of $\hat{L}_z$ and $\hat{H}^{(1)}$:

$$\Phi_{jk}(\mathbf{r}) = \exp\left(\frac{r^2}{2l^2}\right)\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)^j \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)^k \exp\left(-\frac{r^2}{l^2}\right); \;\; l = (\hbar/m\omega)^{1/2}$$

$$E_{jk} = \hbar\omega + \hbar(\omega - \Omega)j + \hbar(\omega + \Omega)k$$

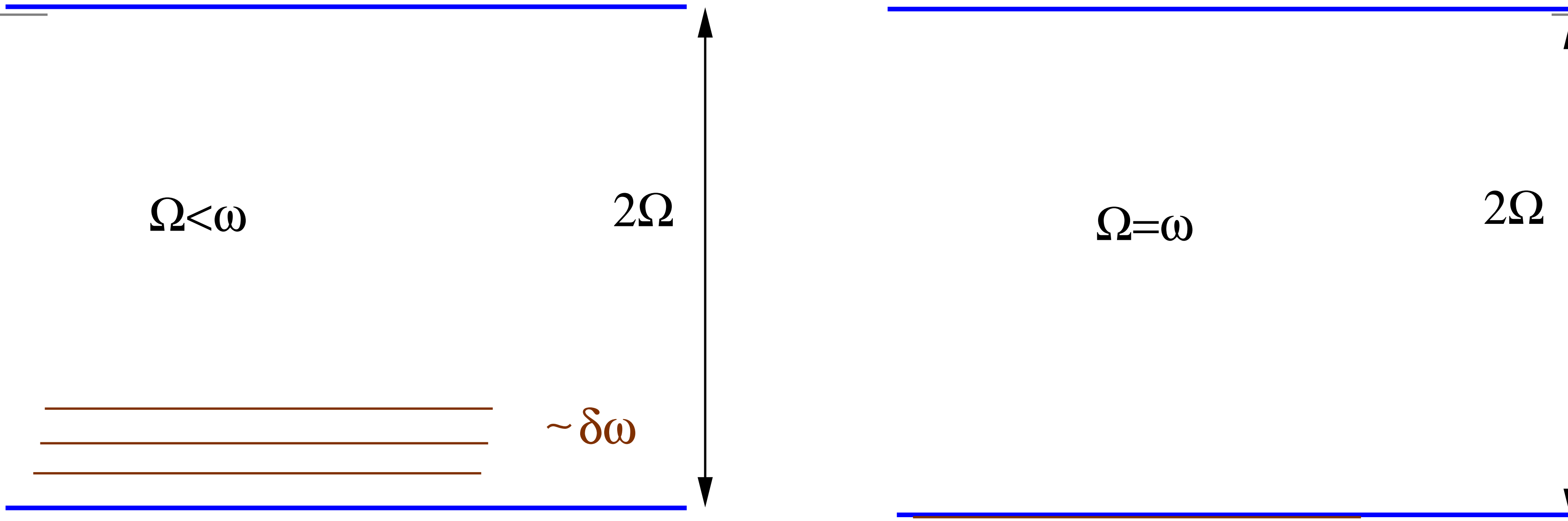
Rapid rotation $\Rightarrow \quad \delta\omega = (\omega - \Omega) \ll \Omega$. Landau levels

$\delta\omega = 0 \Rightarrow$ infinite plane geometry and $E_k = \hbar\Omega(2k+1)$

Level spacing $2\hbar\Omega$. Lowest Landau level (LLL, $k = 0$) $\Rightarrow$

$$\psi(\mathbf{r}) = f(z)\exp\left(-\frac{r^2}{2l^2}\right); \;\; z = x + iy$$

Landau levels



$\omega > \Omega \Rightarrow$ sublevels ($\delta\omega \ll \Omega$)

$$E_{jk} = \hbar\omega + \hbar(\Omega + \omega)k + \hbar\delta\omega j$$

LLL is not degenerate $f_j(z) = \dfrac{z^j}{l^{j+1}\sqrt{\pi j!}}$

$z^j = r^j \exp(ij\phi) \rightarrow$ j is the orbital angular momentum

Rapidly rotating BEC. GP equation

$$T = 0; \quad \delta\omega \ll \Omega; \quad ng \ll \hbar\Omega$$

BEC in the lowest Landau level

$$\psi_0 = \sqrt{n} f(z) \exp\left(-\frac{r^2}{2l^2}\right)$$

GP equation $\Rightarrow$ $$\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{r}} + \frac{m\omega^2 r^2}{2} - i\hbar\Omega\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right) + ng|\psi_0^2 - \mu\right]\psi_0 = 0$$

$$\exp\left(-\frac{r^2}{2l^2}\right)\left[\hbar\delta\omega z\frac{d}{dz} + ngf(z)f^*(\bar{z})\exp\left(-\frac{r^2}{l^2}\right) - \tilde{\mu}\right]f(z) = 0$$

$$\bar{z} = x - iy; \quad \tilde{\mu} =' \mu - \hbar\omega$$

Projection on the LLL $\Rightarrow$ $\hat{P} = \frac{1}{2\pi}\exp(z\bar{z}' - z'\bar{z}'/2)$

$$\hat{P}\psi(z,\bar{z}) = \frac{1}{2\pi}\int \psi(z',\bar{z}')\exp(z\bar{z}' - z'\bar{z}'/2)\,dz'd\bar{z}'$$

$$\hbar\delta\omega z\frac{df(z)}{dz} + \frac{ng}{2\pi}\int f^2(z')f^*(\bar{z}')\exp(z\bar{z}' - 2z'\bar{z}')\,dz'd\bar{z}' - \tilde{\mu}f(z) = 0$$

Vortex lattice

Infinite plane geometry ($\delta\omega = 0$) $\Rightarrow \quad \psi_0(z) = \theta(z)\exp(z^2/2 - z\bar{z}/2)$

Triangular lattice $\theta(z) = (2v)^{1/4}\nu_1(\sqrt{\pi v}z, \tau)$

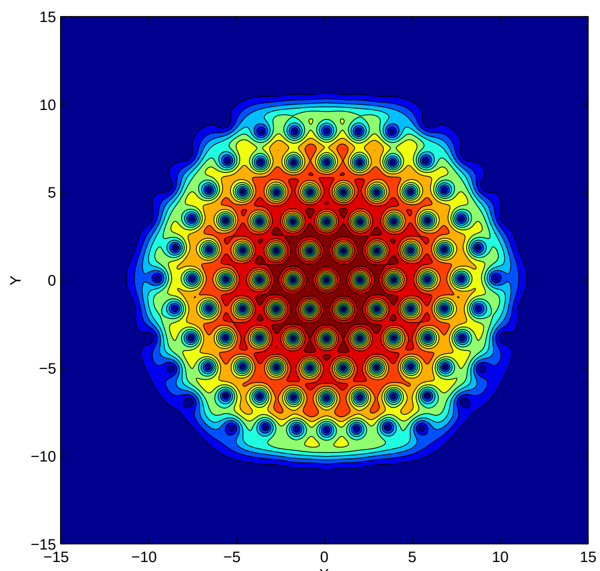
$$\nu_1(\zeta, \tau) = \frac{1}{i}\sum_{n=-\infty}^{\infty}(-1)^n \exp\{i\pi\tau(n+1/2)^2 + 2i\zeta(n+1/2)\}$$

$$\tau = u + iv; \quad v = \sqrt{3}/2; \quad u = -1/2; \quad \tilde{\mu} = \alpha n g; \quad \alpha = 1.1596$$

$\sim l \Rightarrow$ period of the lattice and the size of the vortex core

Numbar of vortices $N_v \sim A/l^2$. Number of particles $N \sim nA$

Criterion of the mean-field regime $N_v \ll N \Rightarrow \; nl^2 \gg 1$



Trapped BEC in the LLL

$\Omega < \omega$ ($\Omega \gg \delta\omega > 0$) Trapped BEC in the LLL

Vortex lattice. Mean-field Quantum Hall regime.

Number of vortices is much smaller than the number of particles

JILA experiment (E. Cornell group) and ENS experiment (J. Dalibard group)

Theoretical studies based on the GP equation and hydrodynamic approach

Last 10 years Ho, Baym, Fetter, Cooper, Aftalion, Dalibard, Sonin, etc.

Is it possible to have rapidly rotating BEC without vortices?

If yes, then how the vortices start to appear?

Quantum transition from zero to one vortex state

State without vortices $\Rightarrow f_0(z) = 1/\sqrt{\pi}$

$$\tilde{\mu}_0 = \frac{ng}{2\pi}$$

,

$$E_0 = \frac{N^2 g}{4\pi}$$

State with one vortex $\Rightarrow f_1(z) = z/\sqrt{\pi}$

$$\tilde{\mu}_1 = \hbar\delta\omega + \frac{ng}{4\pi}$$

$$E_1 = \hbar\delta\omega N + \frac{N^2 g}{8\pi}$$

The state without vortices remains the ground state when $E_0 < E_1 \Rightarrow$

$$\hbar\delta\omega > \frac{ng}{8\pi}$$

Quantum transition from zero to one vortes state

At the point where $\frac{ng}{8\pi} = \hbar\delta\omega$

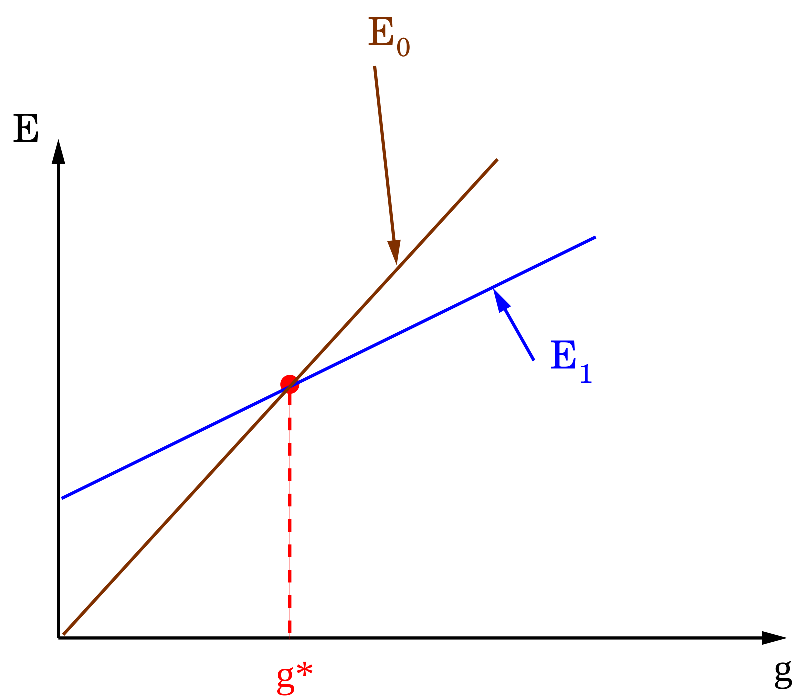
there is a quantum phase transition to the state with one vortex

The chemical potential undergoes a jump $\Rightarrow \tilde{\mu}_1 - \tilde{\mu}_0 = -\hbar\delta\omega$

Quantum phase transition $\Rightarrow$ the transition at $T = 0$

under a change of one of the parameters

Jump in $\mu \Rightarrow$ First order quantum transition



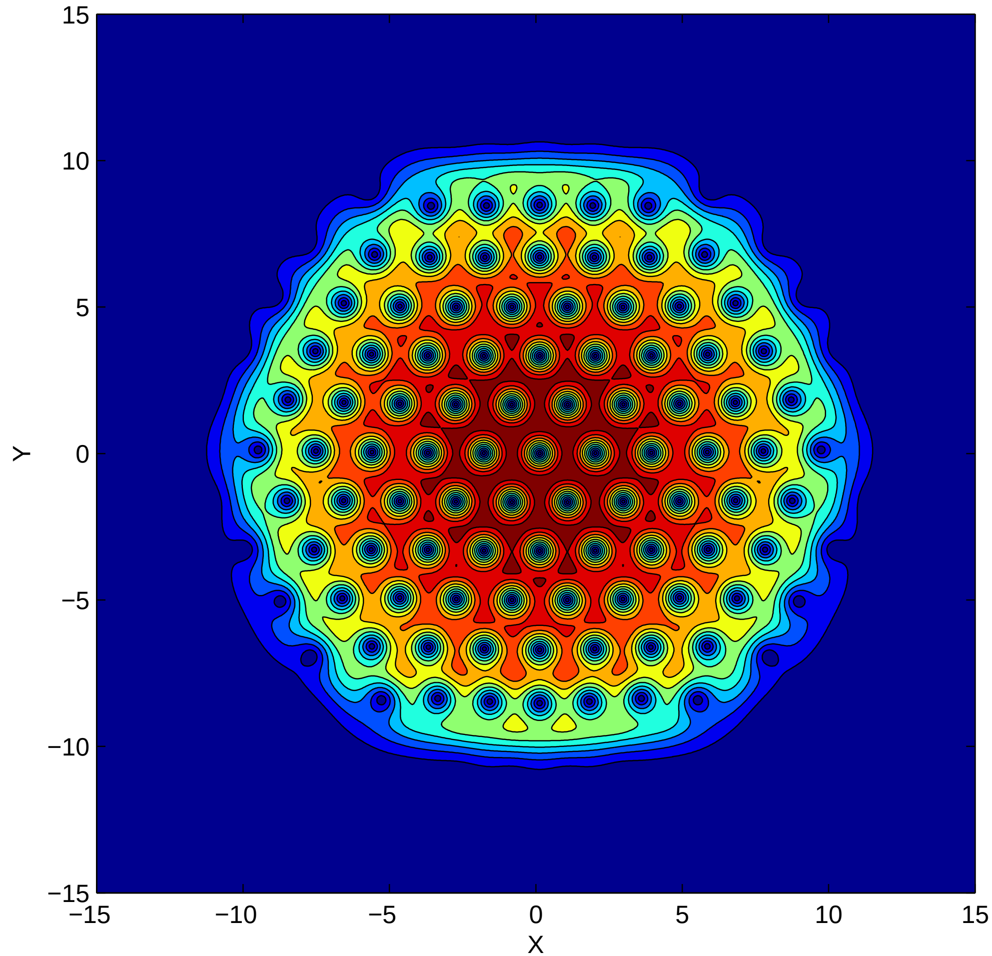
Trapped vortex lattice

How to increase the number of vortices?

Decrease $\delta\omega$ or increase g

Many vortices $\Rightarrow$ trapped vortex lattice

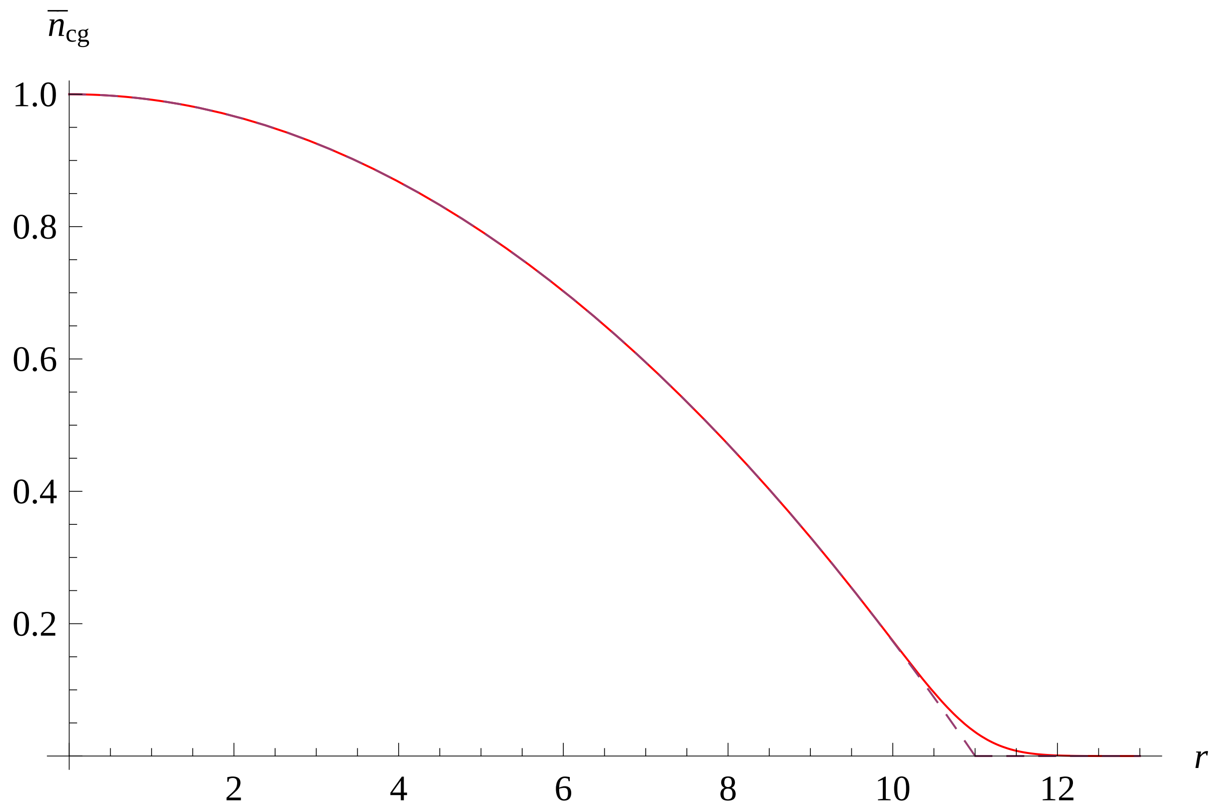
Approximately Thomas-Fermi density profile



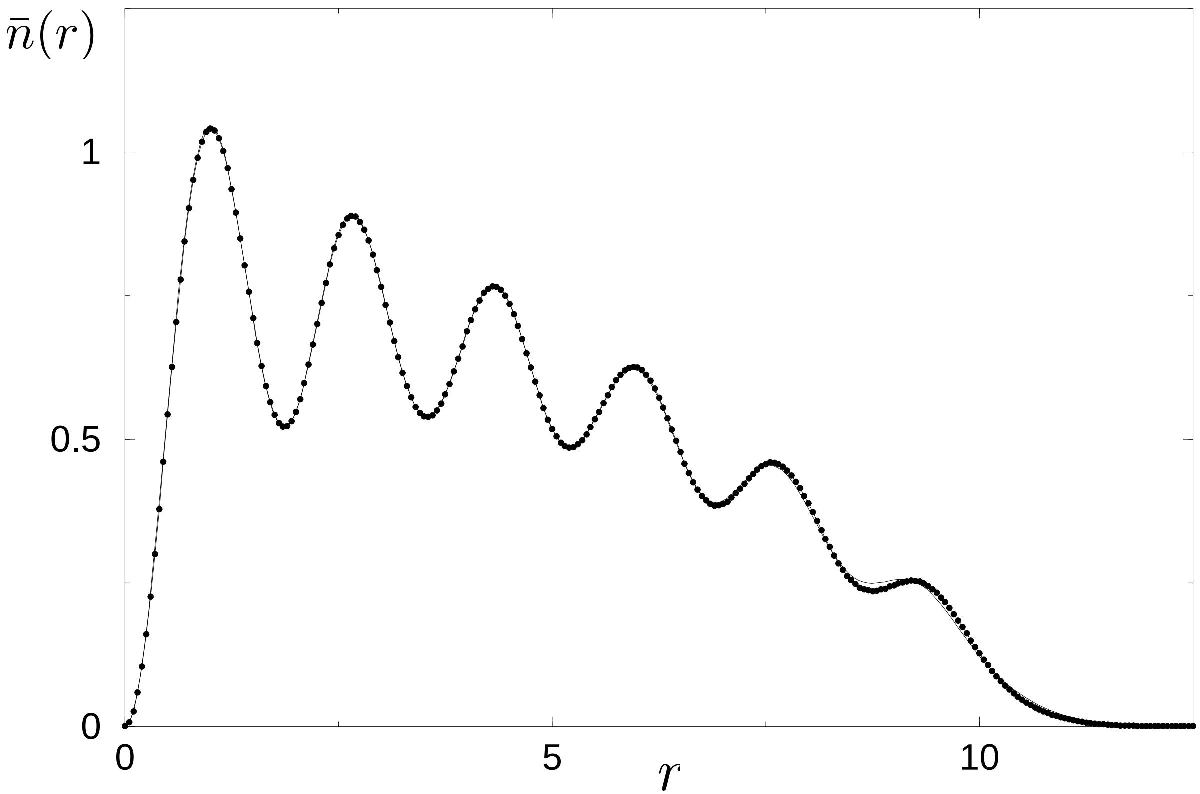
Trapped vortex lattice

Coarse grained density $\bar{n}_{cg} = \bar{n}_{2D}\left(1 - \frac{r^2}{R^2}\right); \quad \bar{n}_{2D} = \frac{2N}{\pi R^2}$

BEC size $R = (2\alpha\beta/\pi)^{1/4} l; \quad \beta = \frac{Ng}{l^2 \hbar \delta\omega}$



Trapped vortex lattice



$$R \sim \left(\frac{Ng}{\hbar \delta \omega} \right)^{1/4} l$$

$$N_v \sim \frac{R^2}{l^2} \Rightarrow \quad \frac{N_v}{N} \sim \left(\frac{g}{N l^2 \hbar \delta \omega} \right)^{1/2}$$

Increase $N_v/N \Rightarrow$ melting of the vortex lattice

Density-phase representation

$$\hat{H} = \int d^2\mathbf{r} \left[-\hat{\psi}^\dagger \frac{\hbar^2}{2m} \Delta_\mathbf{r} \hat{\psi} + \frac{g}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} + V(\mathbf{r}) \hat{\psi}^\dagger \hat{\psi} - \Omega \hat{\psi}^\dagger \hat{L} \hat{\psi} \right]$$

$$i\hbar \frac{\partial \hat{\psi}}{\partial t} = -\frac{\hbar^2}{2m} \Delta_\mathbf{r} \hat{\psi} + g \hat{\psi}^\dagger \hat{\psi} \hat{\psi} + V(\mathbf{r}) \hat{\psi} - \Omega \hat{L} \hat{\psi}$$

$$\hat{\psi} = \exp i\hat{\Phi} \sqrt{\hat{n}}; \quad \hat{\psi}^\dagger = \sqrt{\hat{n}} \exp -i\hat{\Phi}; \quad [\hat{n}(\mathbf{r}), \hat{\Phi}(\mathbf{r}')] = i\delta(\mathbf{r} - \mathbf{r}')$$

$n = n_0(\mathbf{r}) + \delta\hat{n}; \quad \hat{\Phi} = \Phi_0(\mathbf{r}) + \delta\hat{\Phi}$ Small fluctuations of the density

Take zero and linear orders of NLSE with respect to $\delta\hat{n}$ and $\nabla\delta\hat{\Phi}$

Zero order $\Rightarrow$ GP equation for $\Psi_0(\mathbf{r}) = \sqrt{n_0(\mathbf{r})} \exp[i\Phi_0(\mathbf{r})]$

$$-\frac{\hbar^2}{2m} \Delta_\mathbf{r} \Psi_0 + g|\Psi_0|^2 \Psi_0 + V(\mathbf{r}) \Psi_0 - \Omega \hat{L} \Psi_0 = \mu \Psi_0$$

LLL $\Rightarrow$ $\Psi_0 = \sqrt{n_0} f_0(z) \exp(-|z|^2/2)$

Zero order euation

Projected GP equation

$$\hat{P}F(z,\bar{z}) = \frac{1}{\pi}\int dw d\bar{w}\,\exp[-|w|^2 + z\bar{w}]F(w,\bar{w}) \;\Rightarrow \text{LLL}$$

$\Omega = \omega \;\Rightarrow$ geometry of an infinite plane

$$\frac{Ng}{\pi}\int dw d\bar{w} \mathrm{e}^{-2w\bar{w}+z\bar{w}}|f_0(w)|^2 f_0(w) = \tilde{\mu} f_0(z);\; \tilde{\mu} = \mu - \hbar\Omega$$

Triangular vortex lattice $f_0(z) = (2v)^{1/4}\vartheta_1(\sqrt{\pi v}z, q)\,\mathrm{e}^{z^2/2}$

$$q = \exp(i\pi\tau),\; \tau = u + iv,\; v = \sqrt{3}/2,\; u = -1/2$$

$$\tilde{\mu} = \alpha n g; \quad \alpha = 0.1596$$

First order

Linear order $\Rightarrow$ equations for $\delta\hat{n}$ and $\delta\Phi$ $\Rightarrow$

solution in terms of elementary excitations $(u_{\mathbf{k}}, \tilde{v}_{\mathbf{k}})$

$$\delta\hat{n} = \sqrt{n_0}\mathrm{e}^{-|z|^2/2}\sum_{\mathbf{k}}[u_{\mathbf{k}}\exp[-i\Phi_0] - \tilde{v}^*_{\mathbf{k}}\exp[i\Phi_0]]\exp[-i\epsilon_{\mathbf{k}}t]\hat{a}_{\mathbf{k}} + \text{h.c.}$$

$$\delta\hat{\Phi} = \frac{-i\mathrm{e}^{-|z|^2/2}}{2\sqrt{n_0}}\sum_{\mathbf{k}}[u_{\mathbf{k}}\exp[-i\Phi_0] + \tilde{v}^*_{\mathbf{k}}\exp[i\Phi_0]]\exp[-i\epsilon_{\mathbf{k}}t]\hat{a}_{\mathbf{k}} + \text{h.c.}$$

$u_{\mathbf{k}},\ \tilde{v}_{\mathbf{k}} \rightarrow$ solutions of projected BdG equations

$$2g\hat{P}(|\Psi_0|^2 u_{\mathbf{k}}) - g\hat{P}(\Psi_0^2\tilde{v}^*_{\mathbf{k}}) = (\tilde{\mu} + \epsilon_{\mathbf{k}})u_{\mathbf{k}}$$
$$2g\hat{P}(|\Psi_0|^2 \tilde{v}_{\mathbf{k}}) - g\hat{P}(\Psi_0^2 u^*_{\mathbf{k}}) = (\tilde{\mu} - \epsilon_{\mathbf{k}})\tilde{v}_{\mathbf{k}}$$

Solution of projected BdG equations

$$u_{\mathbf{k}} = \frac{c_{1\mathbf{k}}}{\sqrt{S}} f_0 \left(z + \frac{ik_+}{2} \right) \mathrm{e}^{ik_- z/2} \mathrm{e}^{-k^2/4} = c_{1\mathbf{k}} P(f_0 \mathrm{e}^{i\mathbf{kr}})$$

$$\tilde{v}_{\mathbf{k}} = \frac{c_{2\mathbf{k}}}{\sqrt{S}} f_0 \left(z - \frac{ik_+}{2} \right) \mathrm{e}^{-ik_- z/2} \mathrm{e}^{-k^2/4} = c_{2\mathbf{k}} P(f_0 \mathrm{e}^{-i\mathbf{kr}})$$

$$k_\pm = k_x \pm k_y; \quad c_{1\mathbf{k}} = \left[\frac{\tilde{K}(\mathbf{k}) + \epsilon_{\mathbf{k}}}{2\epsilon_{\mathbf{k}}} \right]^{1/2} \mathrm{e}^{k^2/8}; \quad c_{2\mathbf{k}} = \left[\frac{\tilde{K}(\mathbf{k}) - \epsilon_{\mathbf{k}}}{2\epsilon_{\mathbf{k}}} \right]^{1/2} \frac{|K_2(\mathbf{k})|}{K_2(\mathbf{k})} \mathrm{e}^{k^2/8}$$

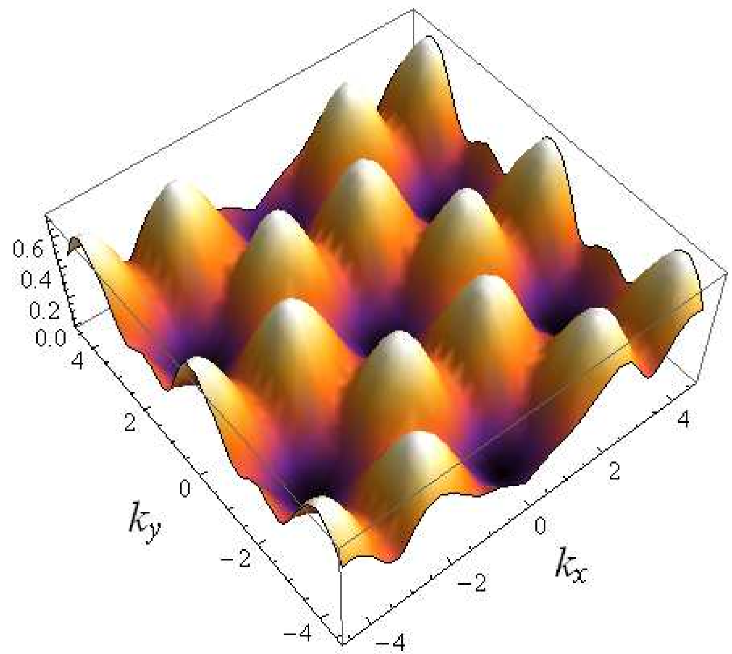
$$\tilde{K}(\mathbf{k}) = 2K_1(\mathbf{k}) - K_1(0; \quad K_1(0) = K_2(0) = \tilde{K}(0) = \alpha \simeq 1.1596$$

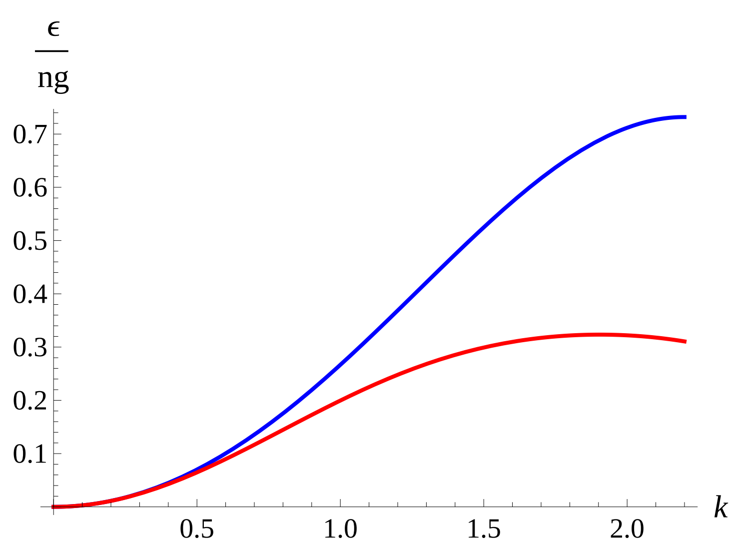
$$K_1(\mathbf{k}) = \sqrt{v} \sum_{n,m=-\infty}^{\infty} (-1)^{nm} \mathrm{e}^{-\pi v (n^2+m^2)} \mathrm{e}^{-\sqrt{\pi v} k_x n + i\sqrt{\pi v} k_y m} \mathrm{e}^{-k_x^2/4}$$

$$K_2(\mathbf{k}) = \sqrt{v} \sum_{n,m=-\infty}^{\infty} (-1)^{nm} \mathrm{e}^{-\pi v (n^2+m^2)} \mathrm{e}^{-\sqrt{\pi v}(k_x - ik_y)(n+m)} \mathrm{e}^{-k_x^2/2 + ik_x k_y/2}$$

Excitation spectrum

$$\epsilon_{\mathbf{k}}^2 = |2K_1(\mathbf{k}) - K_0|^2 - |K_2(\mathbf{k})|^2$$





Low-energy excitations

$$kl \ll 1 \;\Rightarrow\; K_1 = \alpha\left[1 - \frac{k^2}{8} + \frac{(\eta+1)k^4}{64}\right];\; K_2 = \alpha\left(1 - \frac{k^2}{4} + \frac{k^4}{32}\right);\; \eta = 0.8219$$

$$\epsilon = \frac{\alpha\sqrt{\eta}}{4}\, ng(kl)^2 \simeq 0.2628\, ng\,(kl)^2$$

Exactly coincides with Sonin (2005)

Tight confinement in one direction $\Rightarrow g = 2\sqrt{2\pi}\hbar^2 a/ml_0;\; l_0 = \sqrt{\hbar/m\omega_0}$

Rb^{87} $\Omega = 100$ Hz $\omega_0 = 300$ Hz $\Rightarrow ng/\hbar\Omega \simeq 0.1$ at $n \simeq 3\times 10^8$ cm^{-2} (LLL!)

Low-energy excitations $\Rightarrow \epsilon < 1$ Hz

$\nu = \pi n l^2 \gg 1 \;\to$ mean-field regime

Brief historical overview

Elastic oscillations of a vortex lattice in incompressible superfluids

$\Rightarrow \quad \epsilon(k) \propto k$ Tkachenko (1966)

Finite compressibility $\Rightarrow \quad \epsilon(k) \propto k^2$

Volovik/Dotsenko (1979); Baym/Chandler (1983); Sonin (1987)

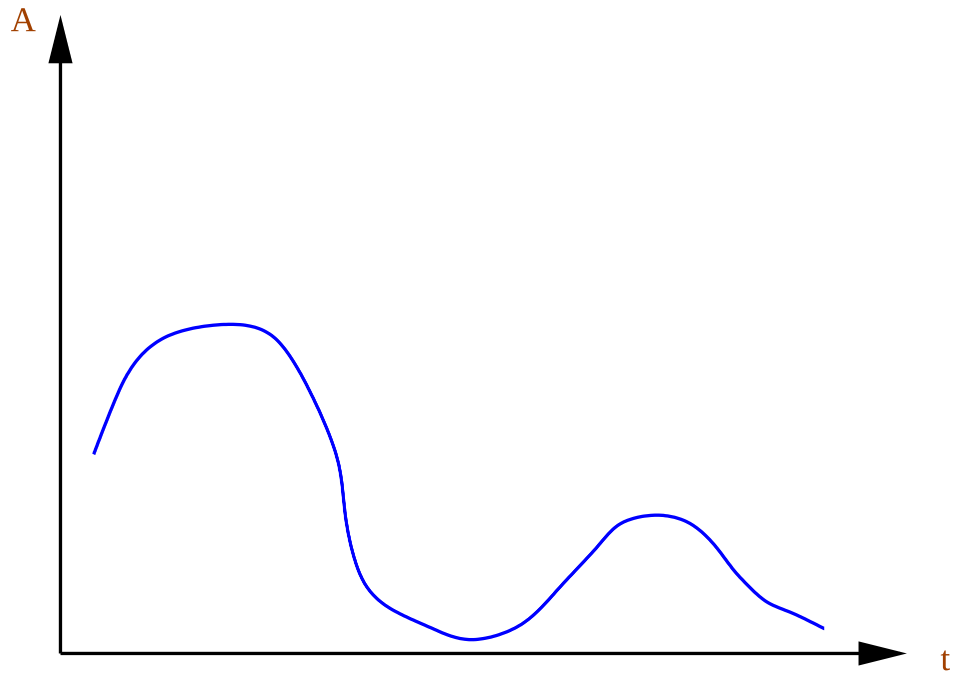
Hydrodynamic approach Baym (2004); Sonin (2005); Fetter

$\omega(k) \propto \sqrt{4\Omega^2 + (s^2 + A)k^2} \Rightarrow$ inertial

$$\epsilon(k) \propto \frac{sk^2}{\sqrt{4\Omega^2 + (s^2 + A)k^2}}$$

Brief historical overview

JILA experiment (E. Cornell group)



Typical picture from the JILA experiment

Calculation of the observed frequencies: Anglin, Baym

Mizushima et al, Bigelow group, Stringari group

Problem

Non-condensed density $n' = \langle \hat{\psi}'^{\dagger} \hat{\psi}' \rangle = \exp(-|z|^2) \sum_{\mathbf{k}} |v_{\mathbf{k}}|^2$

$$n' \sim \int \frac{d^2 k}{\epsilon_{\mathbf{k}}} \sim \int \frac{dk}{k}$$

Low-momentum divergence. No true BEC.

This is not a problem!

One-body density matrix

$$g_1(\mathbf{r}) = \langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(0)\rangle = \Psi_0^*(\mathbf{r})\Psi_0(0)\exp\left\{-\frac{1}{2}\langle(\delta\hat{\Phi}(\mathbf{r}) - \delta\hat{\Phi}(0))^2\rangle\right\}$$

$$\delta\Phi(\mathbf{r}) = -\frac{i}{2}\sum_{\mathbf{k}}\frac{(c_{1\mathbf{k}} + c_{2\mathbf{k}})}{\sqrt{N}}\exp(i\mathbf{kr}/2)\hat{a}_{\mathbf{k}} + h.c.$$

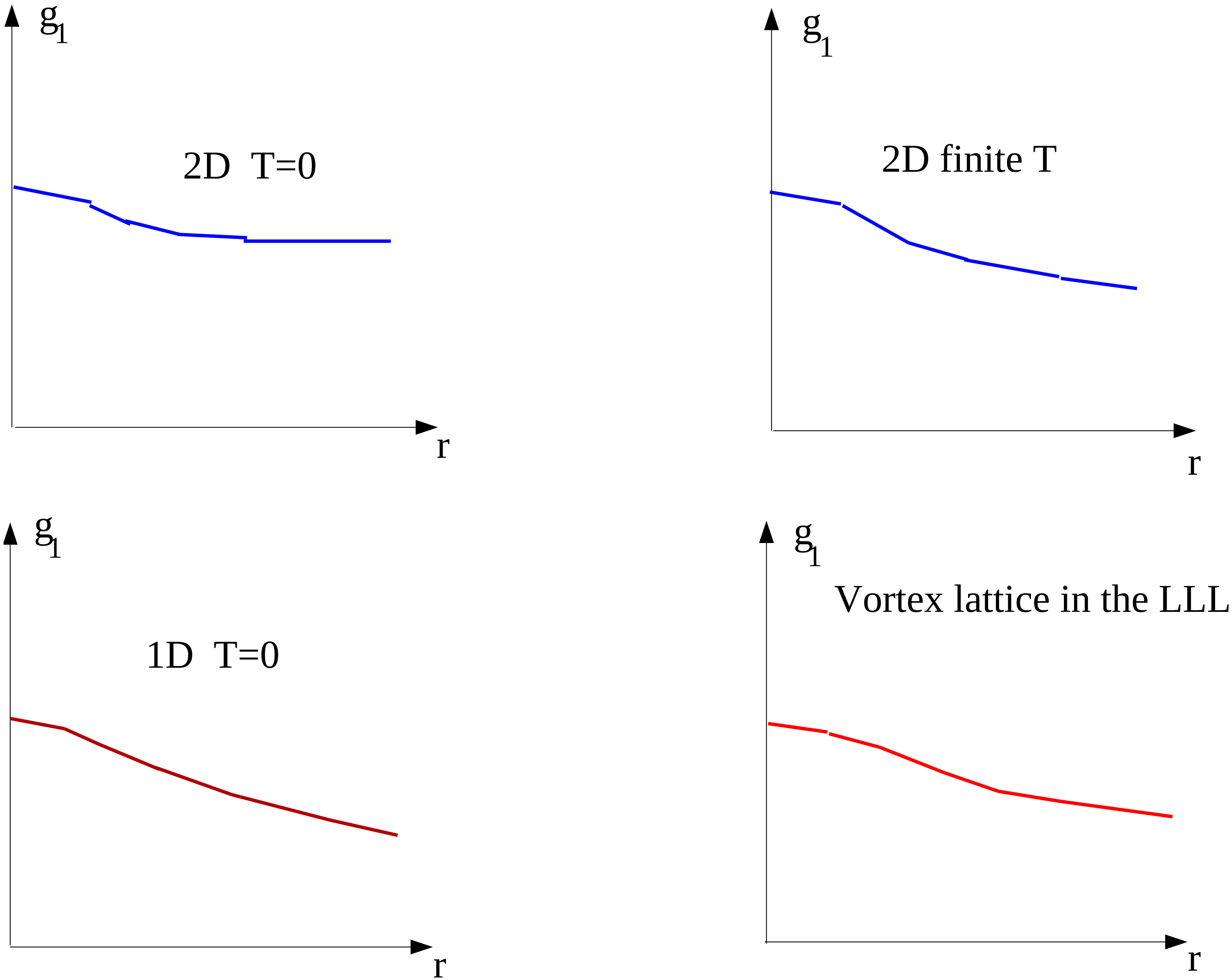
$$\langle(\delta\hat{\Phi}(\mathbf{r}) - \delta\hat{\Phi}(0))^2\rangle = \alpha g\int\frac{d^2k}{(2\pi)^2}\,\frac{(1+2N_k)}{\epsilon_k}[1 - J_0(kr/2)]$$

$$T = 0 \Rightarrow \quad \langle(\delta\hat{\Phi}(\mathbf{r}) - \delta\hat{\Phi}(0))^2\rangle_0 \simeq \frac{2}{\sqrt{\eta}}\,\frac{1}{\nu}\ln\left(\frac{e^C r}{2l}\right)$$

$$g_1(r) \propto \left(\frac{l}{r}\right)^{1/\sqrt{\eta}\pi n l^2}, \quad r \gg l \quad \text{Baym (2004)}$$

Phase coherence length $l_\phi \sim l\exp(nl^2) >>> l$ is extremely large

One-body density matrix



Nature of the vortex state in the LLL

No long-range order. No true BEC in the thermodynamic limit

Algebraic order. QuasiBEC with an extremely large phase coherence length

$\epsilon_{\mathbf{k}} \propto k^2$. No superfluidity

Landau criterion is not satisfied

Analogy with ”flux flow resistivity” in superconductors

Melting of the vortex lattice

One cannot locate the vortex point to a distance smaller than

the mean separation between particles $\Rightarrow (\delta r)^2 \sim \dfrac{1}{2\pi n}$

From the Lindemann melting criterion $\Rightarrow \dfrac{(\delta r)^2}{l^2} \approx 0.02$

we estimate the critical value of the filling factor

(ratio of the number of particles to the number of vortices) $\nu_c = \pi n_c l^2 \sim 10$

More controlled calculation involves the consideration

of collective modes of the lattice. This increases ν_c

Exact diagonalization for a small system gives $\nu_c \approx 6$

Cooper/Wilkin/Gunn (2001)

Strongly correlated states ($\nu < \nu_c$)

Poor understanding for ν just below ν_c

Good understanding for small filling factors

$$\nu = \frac{1}{2} \Rightarrow \text{ Laughlin state}$$

$$\Psi_{Ln}(\{z_i\}) \propto \prod_{i<j}^{N} (z_i - z_j)^2$$

Exact ground state for contact repulsion at $L = N(N-1)$

Ψ_{Lh} vanishes when two coordinates coincide

Zero local two-body correlation

Laughlin state

The average density is uniform. Vortices are not localized in space

Translational symmetry is not broken. Vortices are bound to the particles

Ψ_{Lh} changes phase by $2 \times 2\pi$ when any particle

encircles the position of another particle

Each particle thus experiences 2 vortices

bound to the position of every other particle

The total number of vortices is $N_v = (N-1)$, so that $\nu = N/N_v = 1/2$

Incompressible state with gapped excitations in the bulk and gapless edge modes

Exact diagonalization $\Rightarrow$ incompressibility and the gap of $0.05g/l^2$

Moore-Read state

$$\Psi_{MR} \propto \hat{S}\left[\prod_{i<j\leq N/2}(z_i - z_j)^2 \prod_{N/2<l<m}(z_l - z_m)^2\right]$$

$$\nu = 1$$

Non-abelian statistics for quasiparticle excitations