Strongly interacting quantum gases

Gora Shlyapnikov
LPTMS, Orsay, France
University of Amsterdam

Outline

- Prehistory and Introduction.
- Two-component Fermi gases. Strongly interacting regime
- Molecular BEC regime. Remarkable collisional stability
- Strongly interacting Bose gases
- Stability problem

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Two-component trapped Fermi gas

\[ E_F = \frac{\hbar^2 k_F^2}{2m}; \quad k_F = (3\pi^2 n)^{1/3}; \quad E_F \sim N^{1/3} \hbar \omega \]

Weakly interacting gas \( n|a|^3 \ll 1; \quad k_F|a| \ll 1 \)

\( a < 0 \rightarrow \) Interspecies attraction \( \rightarrow \) Cooper pairing at low \( T \)

Superfluid BCS transition \( \rightarrow T_c \sim E_F \exp\left\{ -\pi / 2k_F|a| \right\} \)

\( T_c \ll 0.1 E_F \quad \text{for ordinary} \ a \quad \text{Very hard to reach} \)
Two-component Fermi gases. Experiments

Dilute limit \( nR_e^3 \ll 1 \)  
Ultracold limit \( \Lambda_T \gg R_e \)

Quantum degeneracy \( \rightarrow \) JILA 1998 \(^{40}\)K

At present \( n \sim 10^{13} - 10^{14} \text{cm}^{-3}; \ T \sim 1 \mu \text{K} \)

Superfluid behavior through vortex formation \( \rightarrow \) MIT
BEC of bosonic molecules \( \rightarrow \) presently in about 10 labs

\[ a > 0 \]

\[ a < 0 \]

BCS

\[ a > 0 \]

weakly bound Molecules
BEC

\[ a < 0 \]

\[ a > R \]

Wide resonance

\[ \varepsilon_0 = \frac{\hbar^2}{ma^2} \]
Feshbach resonance

Bound state of the brown $U(R)$ at the resonance with continuum of green $U(R)$
Strongly interacting regime

\[ T = 0 \quad k_F |a| \gg 1 \quad \rightarrow \quad \text{Only one distance scale} \quad n^{-1/3} \]

Only one energy scale \( E_F \sim \hbar^2 n^{2/3} / m \)

Universal thermodynamics (J. Ho)

Monte Carlo studies \( \rightarrow \mu \approx 0.4E_F \)
(Carlson et al, Giorgini/Astracharchik, etc.)

Theory \( \rightarrow \) Nature of superfluid pairing, Transition temperature, Excitations

Experiments (JILA, MIT, Innsbruck, Duke, ENS, elsewhere) \textbf{Vortices} (MIT)
Vortex lattices

MIT, Zwierlein et al., Science 05

Direct proof of superfluidity!
Positive side of the resonance \((a > 0)\). Gas of bosonic dimers

\[ na^3 \ll 1 \Rightarrow \text{weakly interacting Bose gas} \]

Dimers \(\rightarrow\) The highest rovibrational state \(\Rightarrow\) Remarkable collisional stability

\[ \alpha_{\text{rel}} \sim (k_{\text{eff}} R_e)^2 \sim (R_e/a)^2 \Rightarrow C(\hbar R_e/m)(R_e/a)^s; \quad s = 2.55 \]

\[ \tau \sim (\alpha_{\text{rel}} n)^{-1} \sim \text{seconds} \quad \text{Petrov et al 2003} \)
Bose-Einstein condensates of molecules

Suppressed relaxation  Fast elastic collisions  $a_{dd} = 0.6a$

Efficient evaporative cooling  $\rightarrow$ BEC

The largest diatomic molecules in the world, with the size up to $\sim 3000\text{Å}$

BEC $\Rightarrow$ JILA, Innsbruck, MIT, ENS, Rice, Duke
Strongly interacting Bose gas

Subtle question ⇒ What about BEC at strong interactions?

Experiments (recent several years)

ENS (Salomon group). Stability at a finite $T \gtrsim 1\mu K$ at different $a$ ($^7$Li)

JILA (cornell group). Equilibration at low $T$ with a short lifetime ($^{85}$Rb)
Stability problem

Initarity limit \((a \rightarrow \infty)\). 3-body recombination

\[ A + A + A \Rightarrow A_2 + A + \Delta E \]

Thompson model at \(T = 0\) and \(a \rightarrow \infty\)

Two-component Fermi gas. Recombination to deeply bound states

\[
\frac{1}{\tau_{\text{rec}}} \sim n\sigma v \ast (nR_e^3) \times (kR_e)^2
\]

\[
v \sim \frac{\hbar}{mR_e}; \quad \sigma \sim \frac{1}{k^2}; \quad k \sim n^{-1/3}
\]

\[
\frac{1}{\tau_{\text{rec}}} \sim \frac{\hbar R_e^4}{m} \times n^2 \sim (10 - 100) \text{ s at } n \sim 10^{13} \text{ cm}^{-3}
\]
Stability problem

Bose gas. Recombination to deeply bound states at $T = 0$ and $a \to \infty$

\[ \frac{1}{\tau_{rec}} \sim n \sigma v \ast (nR_e^3) \]

\[ v \sim \frac{\hbar}{mR_e}; \quad \sigma \sim \frac{1}{k^2}; \quad k \sim n^{-1/3} \]

\[ \frac{1}{\tau_{rec}} \sim \frac{\hbar R_e^2}{m} \times n^{8/3} \]

Faster by a factor of $\sim 1/(n^{2/3}R_e^2)$ ($\sim 10^5$ at $n \sim 10^{13}$ cm$^{-3}$)
Stability problem

Large $a > 0$. Weakly bound dimers. Relaxation to deeply bound states

**Two-component Fermi gas**

\[
\frac{1}{\tau_{rel}} \sim \frac{\hbar R_e}{m} \left( \frac{R_e}{a} \right)^{2.5} \times n
\]

**Bose gas**

\[
\frac{1}{\tau_{rel}} \sim \frac{\hbar a}{m} \times n
\]

Faster by a factor of \((a/R_e)^{3.4} (\sim 10^4)\)
Equilibration problem

JILA experiment. Close to unitarity

3-body recombination to a weakly bound state. Very low $T$

$$\frac{1}{\tau_{rec}} \sim \frac{\hbar a^4}{m} \times n^2 \Rightarrow \frac{\hbar}{m} \times n^{2/3}$$

Equilibration rate

$$n a^2 v \rightarrow \frac{\hbar}{m} \times n^{2/3}$$

Equilibration can be faster
Stability of a strongly interacting Bose gas at a finite $T$

ENS experiment. 3-body recombination at a finite $T > 1 \mu K$

Recombination to a weakly bound state $a > 0 \Rightarrow \frac{1}{\tau_{rec}} \sim \frac{\hbar a^4}{m} \times n^2$

Finite $T$. When $\Lambda_T$ becomes comparable with $a$ one replaces $a$ with $\Lambda_T$

$$\frac{1}{\tau_{rec}} \sim \frac{\hbar \Lambda_T^4}{m} \times n^2 \sim \frac{n^2}{T^2}$$

Established in the ENS experiment
Stability of a strongly interacting Bose gas at a finite $T$