Prethermalization in a Non-Equilibrium Many-Body Quantum System

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Non-Equilibrium Systems

Non-equilibrium many-body quantum systems are not well understood.

Equilibrium many-body quantum systems are relatively well understood.
Non-equilibrium many-body quantum systems are not well understood.

How do non-equilibrium systems decay towards the equilibrium state?

Equilibrium many-body quantum systems are relatively well understood.

Non-Equilibrium Systems
Non-Equilibrium Systems: Examples

Spin systems:
Magnetism

Electron transport:
Nano- and molecular electronics

Particle creation and expansion in the early universe:
Baryogenesis, Inflation
Non-Equilibrium Systems: Examples

Spin systems:
Magnetism

Electron transport:
Nano- and molecular electronics

Particle creation and expansion in the early universe:
Baryogenesis, Inflation
Our Non-Equilibrium System

In an ultracold atom system:

A phase-coherently split 1d Bose gas
In an ultracold atom system:

A phase-coherently split 1d Bose gas

1. Prepare an equilibrium 1d Bose gas
In an ultracold atom system:

A phase-coherently split 1d Bose gas

1. Prepare an equilibrium 1d Bose gas

2. Phase-coherently split the gas into two (uncoupled) 1d gases
In an ultracold atom system:

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This is our non-equilibrium system!
Contents

• One-Dimensional Bose Gases
• Creation and Analysis of a Non-Equilibrium System
• History: Previous Experiments
• Results from our New Experiment in Vienna
• Theoretical Description of the Non-Equilibrium Dynamics
• Open Questions
For an effectively 1d Bose Gas (in a trap)

\[ \mu, k_B T \ll \hbar \omega_\perp \]

\[ \omega_\perp \gg \omega_z \]

- Macroscopic occupation of radial ground state
- Longitudinal states populated
1d Bose Gases

- For an effectively 1d Bose Gas (in a trap)
  \[ \mu, k_B T \ll \hbar \omega_\perp \]
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1d Bose Gases

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\[ \mu, k_B T \ll \hbar \omega_{\perp} \]
\[ \omega_{\perp} \gg \omega_z \]

- Macroscopic occupation of radial ground state

- Longitudinal states populated

\[ (e.g. \, \gamma = \frac{mg}{\hbar^2 n} \sim 10^{-2}) \]
1d Bose Gases

Temperature at which thermal phase coherence length is equal to the system size

$$\lambda_T = \frac{\hbar^2 n_{1d}}{mk_B T}$$

We are in here!

(e.g. $T \sim 50 - 100 \text{nK}$)

Degeneracy Temperature

$$N = \frac{k_B T_d}{\hbar \omega_\perp} \ln\left(\frac{2k_B T_d}{\hbar \omega_\perp}\right)$$

Ketterle & van Druten

Small longitudinal density fluctuations

Large longitudinal phase fluctuations

Quasi-condensate Regime
1d Bose Gases

Temperature at which thermal phase coherence length is equal to the system size

\[ \lambda_T = \frac{\hbar^2 n_{1d}}{mk_B T} \]

We are in here!

(e.g. \( T \sim 50 - 100 \text{nK} \))

Degeneracy Temperature

\[ N = \frac{k_B T_d}{\hbar \omega_\perp} \ln(\frac{2k_B T_d}{\hbar \omega_\perp}) \]

Ketterle & van Druten

These longitudinal phase fluctuations are key for our experiments

Small longitudinal density fluctuations

Large longitudinal phase fluctuations

We are in here!
Non-Equilibrium Systems: Examples

In an ultracold atom system:

A phase-coherently split 1d Bose gas

1. Prepare an equilibrium 1d Bose gas

2. Phase-coherently split the gas into two (uncoupled) 1d gases

This is our non-equilibrium system!
1d Bose Gases

Why is this a non-equilibrium system?

1d Bose gas in equilibrium

Phase-coherently split into two 1d gases

Phase profiles are correlated

Difference is only due to quantum shot noise of splitting process

At time $t_h = 0$, $\phi_{nA} \approx \phi_{nB}$

This is our non-equilibrium system!
1d Bose Gases

At time $t_h = 0$, $\phi_{nA} = \phi_{nB}$

At time $t_h = t$, $\phi_{nA} \neq \phi_{nB}$

Non-equilibrium

Equilibrium
(two independent gases with random relative phase)
Non-equilibrium many-body quantum systems are not well understood.

How do non-equilibrium systems decay towards the equilibrium state?

Equilibrium many-body quantum systems are well understood.

At time $t_0 = 0$, $\phi_{nA} = \phi_{nB}$

At time $t = t_f$, $\phi_{nA} \neq \phi_{nB}$

Non-equilibrium

Equilibrium

Time
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Non-Equilibrium Dynamics

- Load ultracold atoms on an atom chip

- Can make long cigar-shaped traps (high aspect ratio)
- Can attain high radial trapping frequencies (several kHz)
- and low axial frequencies (< 5 Hz)

\[ \text{Great for making a single 1d gas!} \]
Non-Equilibrium Dynamics

- Load ultracold atoms on an **atom chip**
- Make an (equilibrium) **1d Bose gas**

\[
T = 120 \pm 30 \text{ nK}
\]
\[
\text{Atom Number} \sim 5000
\]
\[
\lambda_T \sim 3 \mu\text{m}
\]
\[
\omega_\perp = 2\pi \times (2.1 \pm 0.1) \text{ kHz}
\]
\[
\omega_\parallel = 2\pi \times (11 \pm 1) \text{ Hz}
\]
Non-Equilibrium Dynamics

- Load ultracold atoms on an atom chip
- Make an (equilibrium) 1d Bose gas
- Coherently split the 1d gas into two (uncoupled!) 1d gases

Radio-frequency (RF) dressing is used to smoothly form a double-well potential

Non-Equilibrium Dynamics

- Load ultracold atoms on an **atom chip**
- Make an (equilibrium) **1d Bose gas**
- Coherently **split the 1d gas** into two (**uncoupled!**) 1d gases
- Perform **matter-wave interferometry** to extract information from system
Non-Equilibrium Dynamics

- Load ultracold atoms on an **atom chip**
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- Coherently \textit{split the 1d gas} into two (\textit{uncoupled!}) 1d gases
- Perform \textit{matter-wave interferometry} to extract information from system

\[ \text{release from trap} \]

\[ \text{freefall under gravity} \]
Non-Equilibrium Dynamics

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![Diagram](attachment:diagram.png)
Non-Equilibrium Dynamics

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Extracting Information from Interference Patterns
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Select an integration length $L$
Extracting Information from Interference Patterns

Select an integration length $L$

integrated line profile

$atom\ density$

$z$

$x$
Extracting Information from Interference Patterns

\[ f_L(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \left(1 + C_L \cos\left(\frac{2\pi x}{\lambda} + \theta\right)\right) \]
Extracting Information from Interference Patterns

Select an integration length $L$

$\int_{-\infty}^{\infty} \exp \left( -\frac{x^2}{2\sigma^2} \right) \left( 1 + C_L \cos \left( \frac{2\pi x}{\lambda} + \theta \right) \right) dx$

Contrast $C_L$

(over integration length $L$)

Phase $\theta$

(phase of interference pattern with respect to gaussian background)
Extracting Information from Interference Patterns

Select an integration length $L$

$\int_{-\infty}^{\infty} f_L(x) \, dx = \exp \left( -\frac{x^2}{2\sigma^2} \right) \left( 1 + C_L \cos \left( \frac{2\pi x}{\lambda} + \theta \right) \right)$

Contrast $C_L$

Phase $\theta$

(atom density)

(integrated line profile)

(over integration length $L$)

(phase of interference pattern with respect to gaussian background)
Extracting Information from Interference Patterns

Select an integration length $L$

$\left| f_L(x) \right| = \exp \left( -\frac{x^2}{2\sigma^2} \right) \left( 1 + C_L \cos \left( \frac{2\pi x}{\lambda} + \theta \right) \right)$

Contrast $C_L$ (over integration length $L$)

Phase $\theta$ (phase of interference pattern with respect to gaussian background)

integrated line profile

atom density

$\begin{align*}
\text{At time } t_0 = 0, & \quad \phi_{0A} = \phi_{0B} \\
\text{At time } t_L = L, & \quad \phi_{nA} \neq \phi_{nB}
\end{align*}$

Non-equilibrium

Equilibrium

$X$
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History: Previous Experiments, N°.1

- Weiss Lab (Penn State)
- Array of 1d Gases
- Excited two particular momentum modes
- Observed no decay in momentum modes

no thermalisation observed (over several seconds)
Non-equilibrium coherence dynamics in one-dimensional Bose gases

S. Hofferberth, I. Lesanovsky, B. Fischer, T. Schumm & J. Schmiedmayer

Vol 449 20 September 2007 doi:10.1038/nature06149
Measurement of a mean coherence factor $\Psi$

Application of a theory of thermalisation

$\Psi(t) \propto \exp\left(-\frac{t}{t_0}\right)^{2/3}$

This attribution of thermalisation to the data was probably incorrect. Why?

- Measurement of a mean coherence factor $\Psi$
- Application of a theory of thermalisation

$$\Psi(t) \propto \exp \left( -\frac{t}{t_0} \right)^{\frac{2}{3}}$$

Two available theories at the time (2007)

- thermalisation:
  \[ \Psi(t) \propto \exp \left( - \frac{t}{t_0} \right) \]

- dephasing:
  \[ \Psi(t) \propto \exp \left( - \frac{t}{\tau} \right) \]
  Bistritzer & Altman, PNAS 104 24, 9955 (2007)
History: Previous Experiments, N°2

Two available theories at the time (2007):


\[ \Psi(t) \propto \exp \left( -\frac{t}{t_0} \right)^{2/3} \]

Bistritzer & Altman, PNAS 104, 24, 9955 (2007)

\[ \Psi(t) \propto \exp \left( -\frac{t}{\tau} \right) \]
• Two available theories at the time (2007)

**thermalisation**

\[ \Psi(t) \propto \exp \left( -\frac{t}{t_0} \right) \]


**dephasing**

\[ \Psi(t) \propto \exp \left( -\frac{t}{\tau} \right) \]

Bistritzer & Altman, PNAS 104 24, 9955 (2007)
Two available theories at the time (2007)

- Thermalisation
  \[ \Psi(t) \propto \exp\left(-\frac{t}{t_0} \right)^\frac{2}{3} \]

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Our New Experiment in Vienna

• More controlled splitting process
• Ability to measure for longer evolution times
Our New Experiment in Vienna

- More controlled splitting process
- Ability to measure for longer evolution times

Our New Measurements

- Rapid early-time decay
- Long slow decay (consistent with the heating rate in our atom trap)
Our New Experiment in Vienna

- More controlled splitting process
- Ability to measure for longer evolution times

Our New Measurements

- Rapid early-time decay
- Long slow decay (consistent with the heating rate in our atom trap)

From Previous Experiment No. 2
Hofferberth et al.  
Our New Experiment in Vienna

- More controlled splitting process
- Ability to measure for longer evolution times

Our New Measurements

- Rapid early-time decay
- Long slow decay (consistent with the heating rate in our atom trap)

Thermal Equilibrium?
# Beyond the Mean: Full Distribution Functions

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<tr>
<td>15</td>
</tr>
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Beyond the Mean: Full Distribution Functions

\[ X \]

\[ \langle X \rangle \]
Beyond the Mean: Full Distribution Functions

\[ X < X > \]

\[ P(X) \]
Beyond the Mean: Full Distribution Functions

For Equilibrium Systems:

Full quantum distribution of contrast in interference experiments between interacting one-dimensional Bose liquids

VLADIMIR GRITSEV, EHUD ALTMAN, EUGENE DEMLER AND ANATOLI POLKOVNIKOV

Probing quantum and thermal noise in an interacting many-body system

S. HOFFERBERTH, I. LESANOVSKY, T. SCHUMM, A. IMAMBEKOV, V. GRITSEV, E. DEMLER AND J. SCHMIEDMAYER

See also:
• Stimming et al. PRL 105, 015301 (2010) [theory]
• Betz et al. PRL 106, 020407 (2011) [experiment]
Beyond the Mean: Full Distribution Functions

For Equilibrium Systems:

Full quantum distribution of contrast in interference experiments between interacting one-dimensional Bose liquids

Vladimir Gritsev*, Ehud Altman², Eugene Demler¹ and Anatoli Polkovnikov³

We can use this equilibrium theory to investigate our "equilibrium?" state

Theoretical Proposal

Experimental Realisation

Distributions of Interference Contrast $C_L$

We can use this equilibrium theory to investigate our "equilibrium?" state.
Applying Equilibrium Theory

Initial single gas

\[ T = 120 \pm 30 \text{ nK} \]
\[ \lambda_T \sim 3 \text{ } \mu\text{m} \]

Atom Number \( \sim 5000 \)
Applying Equilibrium Theory

Initial single gas

$T = 120 \pm 30 \text{ nK}$

$\lambda_T \sim 3 \text{ \(\mu\)m}$

Atom Number $\sim 5000$

Interference Contrast $C_L$ Distributions

Normalised Squared Contrast, $C^2/\langle C^2 \rangle$
Applying Equilibrium Theory

Initial single gas

\[ T = 120 \pm 30 \text{ nK} \]

\[ \lambda_T \sim 3 \mu\text{m} \]

Atom Number \( \sim 5000 \)

Interference Contrast \( C_L \) Distributions

- FULL CLOUD
- 60 \( \mu \)m
- 38 \( \mu \)m
- 27 \( \mu \)m
- 22 \( \mu \)m

Evolution Time (ms)

12 ms

17 ms

27 ms

Normalised Squared Contrast, \( C^2/<C^2> \)

Compare Equilibrium Theory with Non-Equilibrium Experiment

\[ T = 120 \text{ nK} \]
Applying Equilibrium Theory

Initial single gas

\[ T = 120 \pm 30 \text{ nK} \]
\[ \lambda_T \sim 3 \mu\text{m} \]
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Interference Contrast \( C_L \) Distributions

Compare Equilibrium Theory with Non-Equilibrium Experiment

- \( T = 120 \text{ nK} \)
- \( T = 30 \text{ nK} \)

Normalised Squared Contrast, \( C^2/<C^2> \)
Applying Equilibrium Theory

Initial single gas

\[ T = 120 \pm 30 \text{ nK} \]
\[ \lambda_T \sim 3 \mu \text{m} \]
Atom Number \( \sim 5000 \)

Interference Contrast \( C_L \) Distributions

\[ T = 120 \pm 30 \text{ nK} \]
\[ T = 30 \text{ nK} \]
\[ T_{\text{fit}} = 15 \pm 4 \text{ nK} \]

Compare Equilibrium Theory with Non-Equilibrium Experiment
We now want to compare equilibrium theory to equilibrium experiments.
Measuring Equilibrium Systems

We now want to compare equilibrium theory to equilibrium experiments.
Measuring Equilibrium Systems

We now want to compare equilibrium theory to equilibrium experiments. Equilibrium experiments agree with equilibrium theory.

Normalised Squared Contrast, $C^2/<C^2>$
Equilibrium vs Non-Equilibrium

non-equilibrium distributions
and
equilibrium distributions

$T_{\text{fit}} = 15 \pm 4 \text{ nK}$
Non-Equilibrium Evolution

What happens when we change the initial temperature $T$ of the unsplit system?
Non-Equilibrium Evolution

What happens when we change the initial temperature $T$ of the unsplit system?

EXACTLY the same behaviour

![Graph showing the evolution of Mean Contrast $<C>$ over Evolution Time [ms] for different initial temperatures $T$. The graph includes error bars for each data point. The temperatures plotted are $T = 20 \pm 2 \text{nK}$, $T = 29 \pm 2 \text{nK}$, $T = 39 \pm 4 \text{nK}$, $T = 43 \pm 6 \text{nK}$, $T = 103 \pm 15 \text{nK}$, and $T = 134 \pm 10 \text{nK}$. The y-axis represents the Mean Contrast $<C>$, ranging from 0.2 to 0.6, and the x-axis represents Evolution Time [ms], ranging from 0 to 25. The graph shows that the mean contrast decreases as the evolution time increases for all temperatures, indicating a consistent behavior across different initial temperatures.]
Non-Equilibrium Evolution

What happens when we change the initial temperature $T$ of the unsplit system?

EXACTLY the same behaviour
Recap

track evolution of non-equilibrium state (using matter-wave interference)

BUT this is not a thermal equilibrium state

HOWEVER, it is thermal-like in form

its effective temperature $T_{\text{eff}}$ does not change with initial temperature $T$

AND the initial rapid decay is also independent of the initial temperature $T$

This state represents a fixed point in the evolution of the system!
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Ramsey Interference in One-Dimensional Systems: The Full Distribution Function of Fringe Contrast as a Probe of Many-Body Dynamics

Takuya Kitagawa,¹ Susanne Pielawa,¹ Adilet Imambekov,² Jörg Schmiedmayer,³ Vladimir Gritsev,⁴ and Eugene Demler¹

• Integrable Luttinger-liquid-based theory

\[ H_s = \int \left[ \frac{\rho}{8m} \left( \partial_z \phi_s \right)^2 + g \hat{n}_s^2 \right] dz \]
\[ H_c = \int \left[ \frac{\rho}{8m} \left( \partial_z \phi_c \right)^2 + g \hat{n}_c^2 \right] dz \]

\( \phi_s(z) = \) relative phase \( n_s(z) = \) relative density
\( \phi_c(z) = \) sum phase \( n_c(z) = \) sum density

• Main Concept

The phase fluctuations can be expressed as a sum of \( k \)-modes (momentum modes)

\[ \Delta \hat{\phi}_s (\delta r, t) = \sum_k A_k(t) \sin(k \delta r) \]

where \( k \) is the wave-vector associated with fluctuations of wavelength \( 2\pi/k \)
Theoretical Description of Non-Equilibrium Dynamics


- Integrable Luttinger-liquid-based theory

\[
H_s = \int \left[ \frac{\rho}{8m} (\partial_z \hat{\phi}_s)^2 + g \hat{n}_s^2 \right] dz
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- \( \hat{n}_s(z) = \) relative density
- \( \hat{\phi}_c(z) = \) sum phase
- \( \hat{n}_c(z) = \) sum density

Just after splitting

thermally populated
\[
\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)
\]

populated only by quantum shot noise from the splitting process
\[
\hat{\phi}_s(z) = \hat{\phi}_A(z) - \hat{\phi}_B(z)
\]
Theoretical Description of Non-Equilibrium Dynamics


• Integrable Luttinger-liquid-based theory

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Just after splitting
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\( \hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z) \)
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Equilibrium
thermally populated
\( \hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z) \)
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thermally populated
Theoretical Description of Non-Equilibrium Dynamics


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H_c = \int \left[ \frac{\rho}{8m} (\partial_z \hat{\phi}_c)^2 + g \hat{n}_c^2 \right] dz
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\(\hat{\phi}_s(z)\) = relative phase
\(\hat{n}_s(z)\) = relative density
\(\hat{\phi}_c(z)\) = sum phase
\(\hat{n}_c(z)\) = sum density

This theoretical model describes the dynamics of the non-equilibrium system through the dephasing of the momentum modes \((k\text{-modes})\) of the system, but it does not describe a thermalisation process.

Just after splitting
thermally populated
\[\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)\]
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H_c = \int \left[ \frac{\rho}{8m} (\partial_z \hat{\phi}_c)^2 + g\hat{n}_c^2 \right] dz
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\(\hat{\phi}_s(z)\) = relative phase
\(\hat{\phi}_c(z)\) = sum phase
\(\hat{n}_s(z)\) = relative density
\(\hat{n}_c(z)\) = sum density

This theoretical model describes the dynamics of the non-equilibrium system through the dephasing of the momentum modes (\(k\)-modes) of the system but it does not describe a thermalisation process.

Two-observable FDFs

Just after splitting

thermally populated

\(\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)\)

\(\hat{\phi}_s(z) = \hat{\phi}_A(z) - \hat{\phi}_B(z)\)

populated only by quantum shot noise from the splitting process

Equilibrium

thermally populated

\(\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)\)

\(\hat{\phi}_s(z) = \hat{\phi}_A(z) - \hat{\phi}_B(z)\)

thermally populated
Theoretical Description of Non-Equilibrium Dynamics


- dephasing of the momentum modes (*k*-modes)
  not thermalisation process

\[ f_L(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \left(1 + C_L \cos\left(\frac{2\pi x}{\lambda} + \theta\right)\right) \]

- **Contrast** \( C_L \) (over integration length \( L \))
- **Phase** \( \theta \) (phase of interference pattern with respect to gaussian background)
Theoretical Description of Non-Equilibrium Dynamics


- dephasing of the momentum modes ($k$-modes)
  - not thermalisation process

\[
f_L(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \left(1 + C_L \cos\left(\frac{2\pi x}{\lambda} + \theta\right)\right)
\]

**Contrast** $C_L$

(over integration length $L$)

**Phase** $\theta$

(phase of interference pattern with respect to gaussian background)

Two-observable FDFs

- single measurement
- many measurements
- density plot
Theoretical Description of Non-Equilibrium Dynamics

<table>
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<th>Integration Length, $L$</th>
<th>1.5 ms</th>
<th>2.5 ms</th>
<th>3 ms</th>
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<th>14 ms</th>
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<td>Th.</td>
<td>Th.</td>
<td>Th.</td>
<td>Th.</td>
<td>Th.</td>
<td>Th.</td>
<td>Th.</td>
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<td>18 $\mu$m</td>
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<td>60 $\mu$m</td>
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<td>2.5 ms</td>
<td>3 ms</td>
<td>5 ms</td>
<td>7 ms</td>
<td>10 ms</td>
<td>14 ms</td>
<td>18 ms</td>
<td>23 ms</td>
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### Theoretical Description of Non-Equilibrium Dynamics

#### Integration Length, $L$

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<th>6 $\mu$m</th>
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<th>FULL CLOUD</th>
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<td>23 ms</td>
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</tbody>
</table>

#### Evolution Time

- 1.5 ms
- 2.5 ms
- 3 ms
- 5 ms
- 7 ms
- 10 ms
- 14 ms
- 18 ms
- 23 ms

[Image of the diagram showing different integration lengths and evolution times]
Theoretical Description of Non-Equilibrium Dynamics

- Squared Contrast Distributions

Contrast Squared, $C^2$

Probability Density

Integration Length, $L$

$t_h$

- 2.5 ms
- 4.5 ms
- 7.5 ms
- 12.5 ms
- 17.5 ms
- 27.5 ms
Theoretical Description of Non-Equilibrium Dynamics

Prediction for the effective temperature $T_{\text{eff}}$ [Kitagawa et al. New J. Phys. 13 073018 (2011)]

$$T_{\text{eff}} = \frac{g \rho}{2k_B}$$

- $g = 2\hbar \omega_\perp a_s$
- $\omega_\perp$ — radial trapping frequency
- $a_s$ — scattering length
- $\rho = \text{density}$
Theoretical Description of Non-Equilibrium Dynamics

Prediction for the effective temperature $T_{\text{eff}}$ [Kitagawa et al. New J. Phys. 13 073018 (2011)]

$$T_{\text{eff}} = \frac{g\rho}{2k_B}$$

$g = 2\hbar \omega \alpha_s$

$\rho = \text{density}$

$\omega \alpha_s$ — radial trapping frequency

$\alpha_s$ — scattering length

![Graph showing the relationship between effective temperature $T_{\text{eff}}$ and density.]
Theoretical Description of Non-Equilibrium Dynamics

Prediction for the effective temperature $T_{\text{eff}}$ [Kitagawa et al. New J. Phys. 13 073018 (2011)]

$$T_{\text{eff}} = \frac{g \rho}{2k_B}$$

$g = 2\hbar \omega \bar{a}_s$

$\omega \perp$ — radial trapping frequency

$\bar{a}_s$ — scattering length

$\rho$ = density
Pre-thermalisation

Our system decays to a quasi-stationary state that is not a thermal equilibrium state of the system.

The decay is well described by dephasing to a state that is thermal-like in form (observing the interference contrast $C_L$).

That displays an effective temperature $T_{\text{eff}}$ and rapid initial decay that do not change with initial temperature $T$ and hence is a fixed point in the evolution.


e.g. Moeckel and Kehrein, New J. Phys. 12, 055016 (2010).
Non-equilibrium many-body quantum systems are not well understood.

How do non-equilibrium systems decay towards the equilibrium state?

Equilibrium many-body quantum systems are relatively well understood.

Non-Equilibrium Systems

Evolution Time (ms)

$\langle C^2 \rangle$
dephasing

time
Open Questions

Thermalisation

What is thermalisation?
How does it occur in a 1d system?
It is a subtle process in our case, if it exists at all.

Generalized Gibbs Ensemble (GGE)

It is predicted that the prethermalised state can be described by a GGE.
How can we prove this with an experiment? Can we prove this?

Prethermalization

What is the most general definition of prethermalization?

Thank you.
Experimental References

Relaxation and Prethermalization in an Isolated Quantum System
Science 337, 1318 (2012), arXiv:1112.0013

Prethermalization Revealed by the Relaxation Dynamics of Full Distribution Functions
arXiv:1212.4645

Multimode dynamics and emergence of a characteristic length-scale in a one-dimensional quantum system