Making a spin difference with ultracold gases

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- Expansion dynamics in one dimension
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• **Background: Fermi condensates**

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  • *Is the coexistence of magnetization and superfluidity possible? Is the exotic FFLO (Fulde-Ferrel-Larkin-Ovchinnikov) state stable?*

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Fermi condensates?

- Bose-Einstein condensates are made of bosonic atoms. Other bosons: e.g. photons

- Fermions are constituents of matter: electrons, protons, neutrons, some atoms...

Fermions have to make pairs (which are bosons) to condense to the lowest state
Fermi condensates 2004-2005

BEC-BCS crossover
Related to, e.g., high temperature superconductivity
Science and Physics World ranked the observation of Fermi condensates among the top ten scientific breakthroughs of the year 2004

BEC of molecules (dimers of two Fermions) 2003-2004
\textit{Grimm, Jin, Ketterle, Salomon}

Fermion pairs near the Feshbach Resonance 2004
\textit{Jin, Ketterle}

Density profile throughout the crossover
\textit{Grimm} 2004

Collective modes
\textit{Thomas} 2004, \textit{Grimm} 2004

Heat capacity
\textit{Thomas} 2005

Vortices
\textit{Ketterle} 2005

Pairing gap \textit{Grimm} 2004


J. Kinnunen, M. Rodriguez, and P. Törmä, Science 305, 1131, 2004
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SUPERCONDUCTIVITY IN OUR LIVES

JR Maglev MLX01 – 581 km/h (Japan, 2003)

LHC: Go Higgs!

Future? Superconducting power grid

Long Island, NY
What is superconductivity?

Cooper pairs of spin up and spin down electrons

Not really:

More like:
The Fermi surface

\[ E_F \propto k_F \]

\[ E_F \propto k_x^2 + k_y^2 \]
High-temperature superconductors: complicated structure, spin plays a role (?)" Fermi surfaces (BaKFeAs)

Fe-based superconductors

YBCO

LaOFeAs

Cuprates

Spin fluctuations


Science 328, 441 (2010)
Polarized Fermi gases

Pairing between particles with unequal mass or unequal total number

Related to, e.g., high energy physics (colour superconductivity of quarks)
Spin-Population Imbalanced Fermi Gases

1. Magnetism versus Superconductivity

Chandrasekhar-Clogston limit

Chandrasekhar, APL 1962
Clogston, PRL 1962

critical magnetic field to break superconductivity

2. Exotic superconducting phase?

Fulde-Ferrell-Larkin-Ovchinnikov States

Oscillating order parameter

\[ \Delta \equiv \Delta_0 \exp(iqx) \] (FF)
\[ \Delta \equiv \Delta_0 \cos(qx) \] (LO)

Polarized Superfluid States

fully paired
+ excess unpaired

Sarma, J Phys Chem Solids 1963
Liu & Wilczek, PRL 2003; Sheehy & Radzihovsky, PRL 2006,
Pao, Wu, Yip, PRB 2006; Parish et al., PRL 2007,
Pilati & Giorgini PRL 2008, etc.
Harmonic trap in 3D: The gas is a ball with highest density in the middle.
The effect of the (harmonic) trapping potential

\[ \mu_{\text{eff}} = \mu - V_{\text{trap}}(x) \]
Spin-imbalanced fermions in 3D elongated traps

Experiments:
- Partridge et. al., Science 311, 5760 (2006)
- Shin et. al., PRL 97, 030401 (2006)
- Partridge et. al., PRL 97, 190407 (2006)
- Nascimbene et. al., PRL 103, 170402 (2009)

QMC:

DMFT:

Phase separation; no FFLO
Prediction: FFLO is stabilized in lattices! (mean-field calculation)

Fermi surfaces

Free space

Lattice

\[ q = k_{F\uparrow} - k_{F\downarrow} \]

FFLO in quasi-1D to 3D lattices

D. H. Kim, PT, PRB 85, 180508(R) (2012)
M.O.J. Heikkinen, D.H. Kim, PT, PRB 87, 224513 (2013)
Stronger FFLO signature in (continuum) quasi-1D?

Somewhere between 3D and 1D: an ideal place for FFLO?

In 1D, an exact solution gives FFLO, but no long-range order is possible.

In 3D, the FFLO area may be very narrow.

The long-range order may stabilize FFLO.

Mean-field theory (T=0)
Parish et al., PRL 99, 250403 (2007)

Experiment in 1D (density profiles)
Finite temperature phase diagram
Confirms the mean-field predictions about stability of FFLO in lattices (due to nesting)

Y.L. Loh and N. Trivedi, PRL 104, 165302 (2010)

In contrast, dimensionality does not play a large role, unlike suggested by mean-field: the FFLO is prominent throughout the crossover
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New in ultracold gas experiments: spin-dependent trapping potentials

Different electronic (e.g. hyperfine) states (Bloch, Sengstock, Porto, etc.)

OR

Different fermionic atoms (e.g. $^6$Li, $^{40}$K) (Grimm, Schreck, Hänsch, Dieckmann, Salomon, Walraven, Zwierlein, Inguscio, etc.)

Theory work with spin-dependence/mass-imbalance and fermions e.g. from the groups: Giamarchi, Liu, Wilczek, Zoller, Hofstetter, Demler, Lukin, Stringari, Rozenberg, Dalmonte, Das Sarma, Stoof, Sa de Melo, Batrouni, Scalettar
Pairing in *mixed geometries*

- Pairing with spin-imbalance (mass-imbalance)
  - Chandrasekhar-Clogston limit, FFLO, polarized superfluids, breach pair superfluids, etc.
- Pairing in mixed dimensions
  - A couple of theory papers (Tan, Iskin)
- Our question: pairing in *mixed geometries*?
- Motivation
  - *Spin-dependent confinement*
  - Novel superfluids? High $T_c$? Polarized superfluid?
Our choice of geometry: honeycomb lattice for the up-component, triangular for the down-component

Hubbard-type model, mean-field theory
The non-interacting system
Pairing ansatz at site A

\[ \Delta = U \langle \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} \rangle \]

Bogoliubov transform; then minimize energy to find phases
A new stable polarized superfluid phase: *incomplete breach pair (iBP) state*

Quantum phase transitions (Lifshitz transitions) between states with different Fermi surface topology
Mixed geometry: *multiband* pairing

**a** \( \xi_{\uparrow}(-k) \)

**b** \( \xi_{\uparrow}(k) \)

Energy dispersion (K-\( \Gamma \))

**Noninteracting**

**Interacting**

**c**

**d**

**E=0**

**gapped**

**2-FS**

**e**

**f**

**1-FS**

**2-FS**
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Expansion of a band insulator state

Dynamics of a many-body Fermion system


Core expansion speed as a function of interaction
The system

\[ H = U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} - J \sum_{i,\sigma=\uparrow,\downarrow} c_{i,\sigma}^{\dagger} c_{i+1,\sigma} + h.c. \]

J. Kajala, F. Massel, PT, PRL 106, 206401 (2011)

Earlier 1D dynamics: Kollath, Schollwöck, Zwerger, Heidrich-Meisner, Rigol, Cirac, Zoller, Shlyapnikov, Daley, Santos, Carr, Pupillo, Tezuka, Ueda, Diehl, our group, etc.
Results (TEBD (~ t-DMRG))

\[ \sqrt{n^\uparrow} \quad \text{for} \quad \frac{|U|}{J} = 0.0 \]
$\sqrt{n_{\uparrow}} \text{ for } \frac{|U|}{J} = 1.0$
$\sqrt{n_{\uparrow}}$ for $\frac{|U|}{J} = 10.0$
• With TEBD one can also calculate the doublon density

\[ n_{i \uparrow \downarrow}(t) = \langle \Phi(t) | c_i^{\dagger \uparrow} c_i^{\uparrow} c_i^{\dagger \downarrow} c_i^{\downarrow} | \Phi(t) \rangle \]

• Here we call doublons excitations of the form \( c_i^{\dagger \uparrow} c_i^{\dagger \downarrow} | \emptyset \rangle \)
Left: $\sqrt{n_\uparrow}$, Right: $\sqrt{n_{\uparrow\downarrow}}$, $\frac{|U|}{J} = 10.0$
Experiment by Schneider et al., 2D

Our simulation, 1D

Expansion speed (lattice sites * J)

Interaction |U| (J)
• Our explanation for the numerical findings: describe the two last sites by the Hubbard dimer
• We are interested in the paired state <-> singlet time evolution.
• The singlet state \( \frac{1}{\sqrt{2}} (|\uparrow, \downarrow> - |\downarrow, \uparrow>) \)
• \( n_{\text{Singlet}}(t) = \frac{8}{16 + \frac{U^2}{J^2}} \left( 1 - \cos \left( \sqrt{U^2 + 16J^2t} \right) \right) \)
• Compare to numerics
\[^{n_{\text{Singlet}}}(t)\text{ for } \frac{|U|}{J} = 5.0\]
The frequency comparison.

- **TEBD frequency**
- **Two-site model frequency**
The amplitude comparison.

![Graph showing the comparison between TEBD and Two-site model for the 1st Osc. amplitude against Interaction $|U| (J)$]
The amplitude decay comparison.

![Graph showing the amplitude decay over interaction strength. The graph compares TEBD and a two-site model D. The x-axis represents the interaction strength |U| (J), and the y-axis represents the change of the amplitude. The TEBD line is solid, and the two-site model D line is dashed.]
• In the light of the Dimer analysis, let us take another look at the TEBD results.
\[ \sqrt{n_{\uparrow}} \text{ for } \left| \frac{U}{J} \right| = 10.0 \]

\[ n_{\text{Singlet}}(t) = \frac{8}{16 + \frac{U^2}{J^2}} \left(1 - \cos \left(\sqrt{U^2 + 16J^2t}\right)\right) \]
\[
\sqrt{n_{\uparrow}^{(1)}} \text{ for } \frac{|U|}{J} = 1.0
\]

\[
n_{\text{Singlet}}(t) = \frac{8}{16 + \frac{U^2}{J^2}} \left(1 - \cos \left(\sqrt{U^2 + 16J^2}t\right)\right)
\]
Experiment by Schneider et al., 2D

Our simulation, 1D
Expansion of an FFLO state in a 1D lattice

Inspired by the (continuum) 1D experiment: Liao et al., Nature 467, 567 (2010)

J. Kajala, F. Massel, PT, PRA 106, 206401(R) (2011)
FFLO in 1D lattice

The doublon density as a function of time $n_{\uparrow\downarrow}(t)$. Interaction $|v|/J = 10.0$. 
The unpaired particle density as a function of time $n_\uparrow(t) - n_\uparrow\downarrow(t)$. Interaction $\frac{|V|}{J} = 10.0$. 
Consistent with Bethe ansatz in the large U limit

\[ v_{un}^{\text{max}} = 2J \sin(k_{un}^{\text{max}}) \]
\[ v_{\uparrow\downarrow}^{\text{max}} = 2J \sin(k_{\uparrow\downarrow}^{\text{max}}) \]

We find that \( k_{\uparrow\downarrow}^{\text{max}} = k_{F\downarrow} \)
and \( k_{un}^{\text{max}} = q \).

Therefore, by measuring the maximum expansion velocity of the unpaired particles, one can detect the FFLO momentum. The wavefront corresponding to the maximum velocity is the cloud edge.

\[ q = \arcsin\left(\frac{V_{\text{max}}}{2J}\right). \]
Summary: measuring the expansion velocity of the edge (majority particles) gives the FFLO q-vector!

J. Kajala, F. Massel, PT, PRA 106, 206401(R) (2011)

c.f. C.J. Bolech, F. Heidrich-Meisner, S. Langer, I.P. McCulloch, G. Orso, M. Rigol, PRL 109, 110602 (2012): FFLO correlations lost during the expansion; however, as we point out the initial FFLO q is imprinted to the fastest majority particles that travel at the edge of the cloud.
Dynamics of an impurity in a one-dimensional lattice


c.f. 1D impurity dynamics experiments for bosons: Inguscio group, Bloch group
Conclusions

- Spin-density imbalance and superfluidity
  - The FFLO state stable in lattices at finite $T$ (DMFT)
- Pairing in mixed (spin-dependent) geometries
  - New iBP polarized superfluid state at $T=0$, Lifshitz transitions, multiband character essential
- Expansion dynamics in 1D
  - Band insulator state expansion explained by a simple two-site model
  - FFLO state directly reflected in the expansion velocity of the majority atoms
Dong-Hee Kim (now assistant professor at GIST, South Korea)

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Francesco Massel (now at University of Helsinki)

Jussi Kajala now at Spinverse