Quantum metrology with a scanning probe atom interferometer

Philipp Treutlein
Department of Physics · University of Basel · Switzerland
High-precision measurements of

- Time/frequency
- Inertial forces (gravity, rotations)
- Fine-structure constant
- Atomic properties (polarizability)
- Atom-surface forces (Casimir-Polder)
- Fundamental studies of quantum physics and relativity

Interference → wave nature of matter

Fundamental limit on precision:

Quantum projection noise → particle nature of matter

State-of-the-art interferometers have reached this limit!

Δg/g = 2×10⁻⁸ @ 1 s

tides at Stanford, California


reviews: Cronin et al., Rev. Mod. Phys. 81, 1051 (2009)
Wynands and Weyers, Metrologia 42, S64 (2005)
Quantum metrology

Goal: use entanglement to improve precision measurements

Quantum metrology is useful if N is limited:
- trapped-atom clocks
- atom interferometry with high spatial resolution
- gravitational wave detectors (GEO600, LIGO)

Goal: understand many-particle entanglement
- how to define and quantify it
- different classes of entanglement
- usefulness of entanglement

Standard quantum limit
\[ \Delta \varphi \geq \frac{1}{\sqrt{N}} \]

Heisenberg limit
\[ \Delta \varphi \geq \frac{1}{N} \]
entangled particles

spin squeezing \( \Rightarrow \) entanglement
Outline

Atom interferometers: basic concepts
- Working principle
- Projection noise and standard quantum limit (SQL)
- Spin-squeezing

Experimental setup
- Bose-Einstein condensates on atom chips
- Chip-based clocks and interferometers

Atom interferometer operating beyond the SQL
- Squeezed-state interferometer
- Scanning operation, microwave field imaging

Many-particle entanglement and state tomography
Atomic clocks and interferometers: working principle

**Atomic clock:** measure \( E = h\nu \)

**Atom interferometer:** measure \( E = E_0 + \Delta E(F) \)

accumulated phase after time \( T_R \):
\[
\varphi = \frac{E}{\hbar} T_R
\]

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**Ramsey interferometer**

\[
|0\rangle \rightarrow |0\rangle + |1\rangle \rightarrow |0\rangle + e^{-i\varphi} |1\rangle \rightarrow i\sin\frac{\varphi}{2} |0\rangle + \cos\frac{\varphi}{2} |1\rangle
\]

state preparation
interrogation/phase accumulation
convert phase to amplitude and detect atomic state

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**Ramsey interference fringes**

**Probability** to detect state \( |1\rangle \):
\[
p_1 = \cos^2\left(\frac{\varphi}{2}\right)
\]

---

measurement result for single atom at point of maximum sensitivity (\( \varphi = \pi/2 \)):

**either** \( |0\rangle \) or \( |1\rangle \) \( \Rightarrow \) quantum noise
Ramsey interferometer with N atoms

N independent (non-entangled) two-level atoms

Measure population difference between states $|1\rangle$ and $|0\rangle$:

$$n = \left( N_1 - N_0 \right) / 2$$

Interferometer signal: $\propto N$

Quantum projection noise: $\Delta n \propto \sqrt{N}$

Measurement precision: $\Delta \varphi = \frac{1}{\sqrt{N}}$

$\Rightarrow$ Standard Quantum Limit (SQL)
Collective spin description of interferometer

Atomic internal state: collective spin

\[ N \times \begin{array}{c}
  \bullet \quad |1\rangle \\
  \circ \quad |0\rangle 
\end{array} \quad \Rightarrow \quad \vec{S} = \sum_{i=1}^{N} \vec{s}_i, \quad S = \frac{N}{2} \]

\[ S_z = \frac{1}{2}(N_1 - N_0) \]

Coherent spin state (after \(\pi/2\) pulse):

\[ |\Psi\rangle \sim (|0\rangle + |1\rangle)^N \quad \text{product state} \]

\[ \langle S_x \rangle = \frac{N}{2} \]

\[ \Delta S_z = \Delta S_y = \sqrt{N} / 2 \]

uncertainty of transverse spin components
Ramsey sequence on the Bloch sphere

Wigner function of collective spin on Bloch sphere

Ramsey interference fringes:

\[ \Delta \varphi = \frac{\Delta S_z}{\langle \tilde{S} \rangle} = \frac{1}{\sqrt{N}} \]

Standard quantum limit (SQL)
Spin-squeezing and entanglement generation

Spin squeezing/entanglement through nonlinear dynamics

Kitagawa, Ueda (1993)
Sørensen, Duan, Cirac, Zoller (2001)
...

Squeezing/entanglement parameter (Wineland, 1994):

\[ \xi^2 = 1 \]

\[ H = \kappa S_z^2 \]

\[ \xi^2 < 1 \]

“one-axis twisting”

Time evolution

Squeezing/entanglement parameter (Wineland, 1994):

\[ \xi^2 = \frac{2S(\Delta S_{\theta,\text{min}})^2}{\langle S_x \rangle^2} \]

if \( \xi^2 < 1 \) \( \Rightarrow \)

• useful resource for interferometry beyond standard quantum limit

• atoms entangled

\[ \Delta \phi = \frac{\xi}{\sqrt{N}} \]

Note: not sufficient to reduce spin noise also have to maintain spin coherence (signal)
Ramsey interferometer with squeezed input state

Wigner function of collective spin on Bloch sphere

Ramsey interference fringes:

\[ \Delta \phi = \frac{\Delta S_z}{\langle \tilde{S} \rangle} = \frac{\xi}{\sqrt{N}} \]

Phase uncertainty below SQL (\( \xi < 1 \))
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Many-particle entanglement and state tomography
Atom chips

Micrometer-sized conductor pattern on a chip

Chip-based magnetic microtraps
- trap frequency: up to 1 MHz
- distance to surface: < 1 μm possible

- Bose-Einstein condensation of $^{87}\text{Rb}$
- accurate positioning above surface
- tailored potentials (double well, ...)

BEC
N = 1300
Size ≈ 1 μm
20 μm
Multi-layer microwave atom chips
On-chip preparation of Bose-Einstein condensates

**BEC sequence:**
- mirror-MOT
- optical molasses
- optical pumping
- magnetic trap
- transport atoms
- evaporative cooling to BEC

**all inside the same glass cell**

**pressure:**
$3 \times 10^{-10}$ mbar

**experimental cycle:** 10 s
Clock states of $^{87}\text{Rb}$ in a magnetic trap

Qubit / clock states of $^{87}\text{Rb}$
- both magnetically trappable
- nearly identical potentials
- coherence lifetime: seconds
- limitation: 2-body loss in $|2,1\rangle$
  \(\to\) limit on density/atom number

Rabi oscillations
- fidelity of $\pi$-pulse $> 0.99$
- atom number $N = 1300$
- image both states in one shot
  \(\to\) normalize to total $N$
State-selective absorption imaging

detection noise: $\pm 10$ atoms r.m.s. for $N = 1000$

Ramp (30 ms) to relaxed detection trap (36 Hz/114 Hz), 200 µm from chip, $B_0 = 3.0$ G
Resonant absorption imaging, pulse: 40 µs, $I = 0.8 I_s$, cloud size $15 \times 20 \mu m^2$, $OD_{max} = 1-2$
camera $QE=0.9$, spatial resolution 4 µm
Chip-based atomic clocks and atom interferometers

Atomic clock
compact, portable clock e.g. for satellite navigation


Atom interferometer
• compact, portable inertial sensor
• high spatial resolution (1 μm) imaging of electromagnetic near-fields, surface forces, ...

“interferometric scanning probe”

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Squeezing through state-dependent collisions

\[ H = \chi S_z^2 \]

“one-axis twisting”

Simulation for \( N = 200 \)

How can we implement the squeezing Hamiltonian?

→ collisions between atoms in a state-dependent potential

\[ \chi \approx 0 \]

\[ \chi > 0 \]

\[ \chi \sim a_{00} \int |\phi_0|^4 \, dr^3 + a_{11} \int |\phi_1|^4 \, dr^3 - 2a_{01} \int |\phi_0|^2 |\phi_1|^2 \, dr^3 \]

State-dependent microwave potentials

- BEC of typically $N = 1300 \ ^{87}\text{Rb}$ atoms
- Accurate positioning above the surface
- Coherent control over internal states

Trap:

\[
\begin{align*}
f_x &= 115 \text{ Hz} \\
f_y &= f_z = 520 \text{ Hz} \\
B_0 &= 3.22 \text{ G}
\end{align*}
\]

$41 \mu\text{m}$ from chip surface

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State-dependent microwave potentials

- State-dependent AC Zeeman shift
- Strong near-field gradient

\[ \chi > 0 \]

Spin-squeezed state: Tomography

Variance reduced by
-4.5 ± 0.3 dB
\[ \xi^2 = -4.2 \pm 0.3 \text{ dB} \]
\[ \Rightarrow \text{entanglement} \]
\[ C = 98.5 \pm 0.4 \% \]

R. Schmied et al., New J Phys 13, 065019 (2011)
Squeezed-state interferometer

$|1\rangle$  $|0\rangle$

$S_z^2$  $78^\circ$

$\phi$

$90^\circ$

23.4 ms  $T_R$

State-selective potential

$\pi/2$  $\theta$  $\pi/2$
Squeezed-state interferometer: performance

- Finite Ramsey contrast taken into account
- Technical noise: consistent with shot-to-shot frequency noise
In a Ramsey interferometer with squeezed and coherent input states for varying interrogation times, the interferometer operates at $\xi^2 \approx -4$ dB, below the SQL up to 20 ms. This performance is consistent with the SQL plus independently measured detection noise of $\sigma_{\text{det}} = 3 \times 10^{-3}$ times smaller than the SQL. After $T_R > 5$ ms, both measurements are limited by technical noise, consistent with shot-to-shot fluctuations of $\mu$Hz in the relative frequency between the reference oscillator and the atomic resonator.

After an interrogation time of $5$ ms, our measured noise level corresponds to a single-shot sensitivity in the level shift of $\delta V_{\text{mw}}$. This results in a microwave magnetic field sensitivity of $\delta B = \delta V_{\text{mw}}/\mu B = \mu$T to $\mu$Hz. We emphasize that this sensitivity is achieved with a small probe of $5 \mu$m$^3$ in volume. If we take the probe volume and our experimental cycle of $u$s into account, we obtain a sensitivity of $w \times nT / \mu$m$^3$ Hz.

For comparison, we can scale the results of cm-scale vapor cell rf magnetometers to small volumes, assuming constant density and projection noise-limited performance. State-of-the-art refers to...
measure local microwave field through AC Zeeman level shift

\[ V_{mw}(r) \propto \frac{\mu_B B_{mw}^2(r)}{\hbar \delta} \]
Scanning operation

(b) Phase shift due to microwave near-field

\[ \langle \Delta \varphi \rangle \]

\[ \pi/4 \]

\[ \pi/8 \]

Distance to surface (\( \mu \)m)

(c) SQL

\[ \xi^2 (\text{dB}) \]

\[ 0 \]

\[ -1 \]

\[ -2 \]

\[ -3 \]

\[ -4 \]

State-selective potential

T\( _{mw} \)

Trap position

1 2 3 4 5 6

10 Hz

100 Hz

1 kHz

10 kHz

100 kHz

1 MHz

\( T_S = 23.4 \text{ ms} \)

\( T_R = 20 \text{ ms} \)

\( T_{mw} \)
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Many-particle entanglement and state tomography
**Squeezing implies entanglement**

**Coherent spin state of two atoms:**

\[
|\Psi\rangle = (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) = |00\rangle + |01\rangle + |10\rangle + |11\rangle \quad \text{product state}
\]

**Spin-squeezing:**
reduce amplitude of \( |00\rangle \) and \( |11\rangle \)

\[|\Psi\rangle \rightarrow (|01\rangle + |10\rangle) + \epsilon (|00\rangle + |11\rangle)\]

no longer a product state
⇔ entanglement between atoms

**Spin-squeezing implies entanglement** but not vice versa!

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**Quantum Fisher information**

\[F_Q[\hat{\rho},\hat{J}_n]\]

L. Pezzé and A. Smerzi, PRL 102, 100401 (2009).

Depth of entanglement in the squeezed BEC

Separable state of N atoms:

$$\rho = \sum_k P_k \rho_k^{(1)} \otimes \rho_k^{(2)} \otimes \ldots \otimes \rho_k^{(N)} \quad \xi^2 \geq 1$$

A. Sørensen et al., Nature 409, 63 (2001).

How large are the clusters of entangled atoms in the BEC?

depth of entanglement (minimum block size in $\rho$)

A. Sørensen and K. Mølmer, PRL 86, 4431 (2001).

new data (Basel):

depth of entanglement = 40 ± 10 atoms

Summary

- spin-squeezing of Bose condensate by $\xi^2 = -4.2 \pm 0.3$ dB
- atom interferometer beating the standard quantum limit
- scanning-probe operation for high-resolution field imaging
- data implies depth of entanglement of $40 \pm 10$ atoms

Outlook

- use atom interferometer for measurement of electromagnetic fields near microstructures
- preparation and tomography of other entangled states of BEC
- study many-particle entanglement, quantum Fisher information, ...
The Basel ultracold atoms team

Thomas Lauber  Dr. Roman Schmied  Prof. Philipp Treutlein  Andrew Horsley  Maria Korppi  Andreas Jöckel
Caspar Ockeloen  Dr. Matthew Rakher  Dr. Guan-Xiang Du  Aline Faber

Collaborations: K. Hammerer (Hannover), K. Stannigel, C. Genes, and P. Zoller (Innsbruck)
M. Walter and M. Christandl (ETH Zurich), D. Hunger, S. Camerer, and T. W. Hänsch (LMU Munich)
Yun Li and A. Sinatra (ENS Paris), J. Reichel (ENS Paris), G. Mileti and Ch. Affolderbach (Neuchatel)