Dipolar Interactions and Rotons in Ultracold Atomic Quantum Gases

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Workshop of the RTG 1729
Lüneburg
March 13., 2014
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Dipole-Dipole Interactions

- Dipolar Interactions in magnetic and electric systems

- long ranged and anisotropic

\[ U = \frac{C_{dd}}{4\pi} \frac{1 - 3 \cos^2 \theta}{r^3} \]

- magnetic:
  - \( C_{dd}^B = \mu_0 m^2 \)
  - permanent magnetic dipoles in atoms

- electric:
  - \( C_{dd}^E = 4\pi d^2 \)
  - polar molecules, Rydberg atoms, and light-induced dipoles in atoms

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Realized samples


**Fermion**: $^{53}$Cr  R. Chicireanu, A. Pouderous, R. Barbé, B. Laburthe-Tolra, E. Maréchal, L. Vernac, J.-C. Keller, and O. Gorceix, PRA 73, 053406 (2006)

**Boson**: $^{87}$Rb  M. Vengalattore, S. R. Leslie, J. Guzman, and D. M. Stamper-Kurn, PRL 100, 170403 (2008)

**Boson**: $^{164}$Dy  M. Lu, N. Burdick, S. Youn, and B. Lev, PRL 107, 190401 (2011)


**Fermion**: $^{167}$Er  Aikawa et al., arXiv:1310.5676 (2013)
Erbium: Collisional properties

- producing BEC and degenerate Fermi gases of Erbium
- complicated collisional properties → lot of Feshbach resonances

- lot of Feshbach resonances within small $B$-range → better control of contact strength
Cooling of Fermions

- No s-wave collisions in spin-polarized Fermi gases
- Without dipole-dipole interaction no direct evaporative cooling
- Strong dipole-dipole interactions lead to thermalization through p-wave interactions
- Very small inelastic collisions due to large p-wave barrier

Dipole-dipole interaction is partially attractive → stability depends on contact interaction and geometry

What is a Roton?
Calculation of the Excitation Spectrum

• weak interacting limit $\rightarrow$ most of particles stay in condensate

• description by Gross-Pitaevskii Equation

$$\imath\hbar \dot{\psi} = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(x) + g|\psi|^2 + \int d^3r' U(r - r')|\psi(r', t)|^2 \right] \psi$$

• Bogoliubov approximation

$$\psi(r, t) = \psi_0(r, t) + \delta\psi(r, t)$$

• linear transformation

$$\delta\psi(r, t) = u(r)e^{-\frac{i}{\hbar}(\mu + \hbar\omega)t} + v^*(r)e^{\frac{i}{\hbar}(-\mu + \hbar\omega)t}$$
Calculation of the Excitation spectrum

\[
\begin{align*}
\left[ -\frac{\hbar^2 \nabla^2}{2m} + V(x) + 2g |\psi_0(r)|^2 + \int d^3 r' U(r - r') |\psi_0(r')|^2 - \mu \right] u \\
+ g\psi_0^2 v + \psi_0 \int d^3 r' U(r - r') \left[ \psi_0(r') v(r') + \psi_0^*(r') u(r') \right] = \hbar \omega u \\
\left[ \frac{\hbar^2 \nabla^2}{2m} - V(x) - 2g |\psi_0(r)|^2 - \int d^3 r' U(r - r') |\psi_0(r')|^2 + \mu \right] v \\
- g(\psi_0^*)^2 u - \psi_0^* \int d^3 r' U(r - r') \left[ \psi_0(r') v(r') + \psi_0^*(r') u(r') \right] = \hbar \omega v
\end{align*}
\]

- homogeneous solution

\[
\hbar \omega = \pm \sqrt{\frac{\hbar^2 k^2}{2m} \left( \frac{\hbar^2 k^2}{2m} + 2gn_0 + 2\hat{U}(k)n_0 \right)}
\]
Experiments and General Properties

What is a Roton?

Our Work

Summary

Excitation spectrum

Excitation Spectra

Roton Confinement

Excitation Spectrum for 2D homogeneous system

- linear dependence for small momenta
- quadratic behavior for large momenta
- roton like excitation spectrum for pancake traps
- if \( k \ll \frac{1}{L} \) two dimensional excitations → particles repel each other → excitations are phonons
- for \( k \gg \frac{1}{L} \) excitations acquire 3D character → interparticle repulsion is reduced

Inhomogeneous System and Fingers

- Cylinder-symmetric system → $z$-projection of angular momentum is "good" quantum number
- Emerging roton fingers
Roton Confinement

- depth of roton minimum is interaction dependent
- interaction is density dependent

\[ E_{n,m} = \sqrt{\epsilon_0^2 + An + Bm^2} \]

What is the difference between a Roton and a Phonon?

- Phonon mode extends over condensate size
- Roton mode is localized around center
- Condensate-excitation interaction attractive at short wavelengths

Detection of Rotons

- no observation yet
- analyzing the structure factor of trapped dipolar gas
- stability spectroscopy
- atom number fluctuations can reveal presence and locality of rotons
- problem locality → can be solved by using box potential

Rotons are confined at high density regions

Large disturbance of density profile due to rotons

At which point is the GPE approach not valid anymore?

\[ N_0 \gg N_D \] but not \( n_0(r) \gg n_D(r) \) for all \( r \)
Solve Bogoliubov-de-Gennes Equations for full 3-dimensional system

\[
-i\hbar \dot{\psi} = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(x) + g|\psi_0|^2 + \int d^3 r' U(r - r') |\psi_0(r')|^2 \right] \psi_0
\]

\[
\left[ -\frac{\hbar^2 \nabla^2}{2m} + V(x) + 2g|\psi_0(r)|^2 + \int d^3 r' U(r - r') |\psi_0(r')|^2 - \mu \right] u
\]

\[
+ g\psi_0^2 v + \psi_0 \int d^3 r' U(r - r') \left[ \psi_0(r') v(r') + \psi_0^*(r') u(r') \right] = \hbar \omega u
\]

\[
\left[ -\frac{\hbar^2 \nabla^2}{2m} - V(x) - 2g|\psi_0(r)|^2 - \int d^3 r' U(r - r') |\psi_0(r')|^2 + \mu \right] v
\]

\[
-g(\psi_0^*)^2 u - \psi_0^* \int d^3 r' U(r - r') \left[ \psi_0(r') v(r') + \psi_0^*(r') u(r') \right] = \hbar \omega v
\]
The Bogoliubov-de-Gennes Equations

- BdG are linear set of equations
  \[ M(r)\bar{u}_i(r) = E_i\bar{u}_i(r) \]
- Number of matrix elements scales as \( N^2 \) for every direction
- Change to harmonic oscillator basis (\( \phi_i \))
  \[ M_{ij} = \int dx \phi_i M \phi_j \]
- Calculate condensate depletion \( n_D \)
  \[ n_D(r) = \langle n_p \rangle (u(r)^2 + v(r)^2) + v(r)^2 \]
- Investigate quantum and thermal depletion, local form of thermal cloud, effects on superfluidity etc.
Feshbach resonances in Erbium at small magnetic fields → very good control of contact interaction
Direct evaporative cooling in Erbium due to large magnetic interactions
Roton like excitation spectrum in dipolar systems with pancake geometries
Rotons are confined to large density regions and cause large density disturbances
We will investigate validity and consequences of rotons
Thanks for your attention!