“Universal” vs “Non-Universal” The Cs case

A. Zenesini, M. Berninger, B. Huang, S. Besler, H.-C. Nägerl, F. Ferlaino, and R. Grimm
UNIVERSITY OF INNSBRUCK, AUSTRIA

IN COLLABORATION WITH
J.M. HUTSON & P.S. JULIENNE
J. VON STECHER & C. GREENE
Triiton denotes the reduced mass of a single neutron and its contribution, the spherical part of the nuclear radius (\(5.4 \text{ fm}\)) are due to a deformation effect. The spherical part of the nuclear radius is increased by a factor of 2.22 fm so as to reproduce the present results. The filled square and circles show the present results for isotopes. The filled square and circles show the present results.

\[
\text{PRL 104, 062701}
\]

Cold atoms and droplets

\[
\text{J. ROBINSON WEBSITE}
\]

Cold atoms and droplets
Triton

where the nuclear shapes are parameterized as rotational

We fixed

was

noted that both results overlap within their error bars. The

\( r_{n} \) and those determined at GSI \[ 7 \]

\( /C_{26} \) does not follow the systematic behavior of radii in carbon

\( /C_{6} \)

\( /C_{6} \)

\( /C_{30} \)

\( /C_{20} \)

The square of the Yukawa function is known to be a

\[ m \]

\( /C_{60} \)

\( /C_{90} fm \)

\( /C_{10} \)~

\( /C_{10} \)

\( /C_{25} \)

\( /C_{50} \)

\( /C_{14} \)

\( /C_{10} \)

\( /C_{14} \)

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\( /C_{25} \)
3... IS THE MAGIC NUMBER...

Can universality explain all of them?
EFIMOV TRIMERS
EFIMOV TRIMERS
EFIMOV TRIMERS
EFIMOV TRIMERS
EFIMOVO TRIMERS

\[ R \]

\[ \alpha \]
EFIMOV TRIMERS

SOVIET JOURNAL OF NUCLEAR PHYSICS

WEAKLY-BOUND STATES OF THREE RESONANTLY-INTERACTING

V. N. EFIMOV

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
Submitted February 16, 1970
Yad. Fiz. 12, 1080–1091 (November, 1970)

It is shown that if the pair forces of three identical particles are sufficient bound states of low energy is produced. The quantum numbers of all the spinless bosons 0 and for nucleons 1/2, T = 1/2. The dimension of the size radius of the pair forces. The most favorable conditions for the appearance occur for three spinless neutral bosons: the conditions are less favorable and particles with spin and isospin. The possibility of existence of such particles (in the C$^{12}$ nucleus) and of three nucleons (H$^3$) is considered.
THE ENERGY LANDSCAPE OF UNIVERSAL TRIMERS

\[ E_{3B} = E_{3B}(a, \Phi) \]

\[ a < 0 \quad \text{energy} \quad a > 0 \]

HALO DIMER
THE ENERGY LANDSCAPE OF UNIVERSAL TRIMERS

\[ E_{3B} = E_{3B}(a, \Phi) \]

\[ a < 0 \quad 22.7 \quad \text{energy} \quad a > 0 \]

HALO DIMER
THE ENERGY LANDSCAPE OF UNIVERSAL TRIMERS

\[ E_{3B} = E_{3B}(a, \Phi) \]

ONE PARAMETERS AND THE COMPLETE SPECTRUM IS DEFINED
3... IS THE MAGIC NUMBER...

CAN UNIVERSALITY EXPLAIN ALL OF THEM?
INVESTIGATION WITH COLD ATOMS

\[ a(B) \]

- Loss minimum
- Loss maximum

Energy

\[ a < 0 \quad \text{or} \quad a > 0 \]

\[ a_\pm \]

2nd ET

1/515

1st ET

22.7
INVESTIGATION WITH COLD ATOMS

$a(B)$

$a < 0$  
$a > 0$

$a_-$  
$1/1515$

Energy

2nd ET

1st ET

Loss minimum

Loss maximum
INVESTIGATION WITH COLD ATOMS

Feshbach Resonance

\(a(B)\)

\(a < 0\)

\(a > 0\)

\(1/\alpha\)

Energy

\(a_{-}\)

\(22.7\)

1st ET

1/515

2nd ET

\(-\) Loss minimum

\(\oplus\) Loss maximum

\(\bar{a}\)

\(a_{+}\)
INVESTIGATION WITH COLD ATOMS

INVESTIGATION WITH COLD ATOMS

Li7 - E. Pollack et al, Science 326, 1683 (2009)
Fermions and mixtures...

3BP NOT TUNABLE BUT...

Diagram showing energy levels and transitions with labels such as $a_-$, $a_+$, and $a_*$, indicating first and second electron transfers (ET).
3BP NOT TUNABLE BUT...
CS Feshbach Resonances

CS Feshbach Resonances

Magnetic Field (G)

$\frac{a}{10^3 a_0}$

$(4,4)6$  $(3,4)7$  $(3,4)6$  $(F_1,F_2)F$
CS Feshbach Resonances

\[ a(10^3 a_0) \]

Magnetic Field (G)

(F1,F2)F

\[ (4,4)6 \]

\[ (3,4)7 \]

\[ (3,4)6 \]
Our Method:

1) Feshbach scan (d,g,i): poles & zero-crossings
2) Molecular binding energy measurements: $E_B$
3) Convolution of coupled-channel calculations and bound-state calculations (intense collaboration with P. S. Julienne and J. Hutson)

- The Efimov physics relies on $\alpha$
- In the Lab, we measure $B$

M. Berninger, A. Zenesini et al., Feshbach Spectroscopy, Feshbach Resonances and Coupled-Channel Potentials for Cesium Molecules at High Magnetic Field, in preparation.

A. Zenesini et al., Creation of Bose Einstein Condensates of Cesium at High Magnetic Fields, in preparation.
MANY EFIMOV RESONANCES,
ONLY ONE VALUE...

MEASURING THE BROTHERS POSITIONS

<table>
<thead>
<tr>
<th>FR</th>
<th>(a_0 (a_0))</th>
<th>(\eta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s-wv</td>
<td>-871(22)</td>
<td>0.10(3)</td>
</tr>
<tr>
<td>(s/g)-wv</td>
<td>-1029(58)</td>
<td>0.12(1)</td>
</tr>
<tr>
<td>(s/g)-wv</td>
<td>-957(80)</td>
<td>0.19(2)</td>
</tr>
<tr>
<td>s-wv</td>
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<td>0.08(1)</td>
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</table>

![Graph showing recombination length vs. scattering length](image-url)
MEASURING THE BROTHERS POSITIONS

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the $y$-scale covers one Efimov period

![Graph showing recombination length vs. scattering length (10^3 a_0)]
MEASURING THE BROTHERS POSITIONS

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The y-scale covers 1/10 of the Efimov period.
The 3BP is largely insensitive to $B$. 

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The $y$-scale covers 1/10 of the Efimov period.
WHY ON A LINE?

\[ \frac{\alpha_T^*}{R_{vdW}} \]

Fig. from arXiv:1111.1484: CHIN

NOTED FOR THE FIRST TIME IN THE CASE OF LITHIUM AND CESIUM BY KHAYKOVICH
PRL 105, 103203 (2010).
WHY ON A LINE?

$\frac{a_T^*}{R_{vdW}}$ ~ $9.1(2) R_{vdW}$

Fig. from arXiv:1111.1484: CHIN

$^{133}$Cs  $^7$Li  $^6$Li  $^{85}$Rb  $^{39}$K

NOTED FOR THE FIRST TIME IN THE CASE OF LITHIUM AND CESIUM BY KHAYKOVICH
PRL 105, 103203 (2010).
IS IT A KIND OF QUANTUM REFLECTION?

\[ a_T^* \sim 9.1(2) R_{VdW} \]

EX.
IS IT A KIND OF QUANTUM REFLECTION?

\[ a_T^* \sim 9.1(2) R_V dW \quad \varepsilon \text{X.} \]
IS IT A KIND OF QUANTUM REFLECTION?

\[ a^*_T \sim 9.1(2) R_V dW \]
IS IT A KIND OF QUANTUM REFLECTION?

\[ a_T^* \sim 9.1(2) R_V dW \]
IS IT A KIND OF QUANTUM REFLECTION?

\[ a_T^* \sim 9.1(2) R V_d W \]

\[ a_T^* \sim 9.48(2) R V_d W \]

\[ \sim 9.73(2) R V_d W \]

arXiv:1111.1484: CHIN
arXiv:1201.1176; Wang et al
arXiv:1201.4310: SCHMIDT (MFT)
IS IT A KIND OF QUANTUM REFLECTION?

\[
a_T^* \sim 9.1(2) R_{vdW} \\
la_T^* \sim 9.48(2) R_{vdW} \sim 9.73(2)
\]

arXiv:1111.1484: CHIN
arXiv:1201.1176; Wang et al
arXiv:1201.4310: SCHMIDT (MFT)
A. Zenesini et al, *Resonant Five-Body Recombination in an Ultracold Gas*, Submitted
A. Zenesini et al, *Resonant Five-Body Recombination in an Ultracold Gas*, Submitted
3... IS THE MAGIC NUMBER...

CAN UNIVERSALITY EXPLAIN ALL OF THEM?
LI, K, CS, RB, RB+CS

- Loss minimum
+ Loss maximum
$\text{LI, K, CS, RB, RB+CS}$

$\text{LI, K, CS}$

$a_-$

2nd ET

$1/515$

1st ET

Loss minimum

Loss maximum

$22.7$

Energy
LI, K, CS, RB, RB+CS

\[ a_+ \quad 22.7 \quad a_- \]

Energy

\[ \bar{a}_* \]

Loss minimum

Loss maximum

LI, CS

LI, K, CS
1st ET

2nd ET

LI, K, CS, RB, RB+CS

LI, K, CS

Loss minimum

Loss maximum

a_ < a

1/515

22.7

1/a

a+

a*

a

LI, CS

1st ET

2nd ET
$LI, K, CS, RB, \text{ RB+CS}$

Loss minimum

Loss maximum

1st ET

2nd ET

$\frac{1}{515}$

$\frac{1}{a}$
A-D IN CESIUM: I° ACT

A-D IN CESIUM: I° ACT

A-D IN CESIUM: 1° ACT

universal relation:

\[ \frac{a_{*}^{(n+1)}}{a_{*}^{(n)}} \]

theory 1.06
experiment 0.47

A-D IN CESIUM: 2° ACT

binding energy $E_b$ (MHz)

magnetic field strength $B$ (G)
A-D IN CESIUM: 2° ACT

binding energy $E_b$ (MHz)

magnetic field strength $B$ (G)
A-D IN CESIUM: 2° ACT

binding energy $E_b$ (MHz) vs. magnetic field strength $B$ (G)
A-D IN CESIUM: 2° ACT
A little more complicated

binding energy $E_b$ (kHz)

magnetic field $B$ (G)

Cesium Model 2012
...BUT REASONABLE...

THE COUPLING IS **100KHZ**

WITH **SIX** QUANTA OF ANGULAR MOMENTA!!!
A-D IN CESIUM

\[ \beta (\text{cm}^3/\text{s}) \]

\[ \text{scattering length (a)} \]

Graph showing data points labeled as HF and LF.
A-D IN CESIUM

![Graph showing scattering length versus beta (cm^3/s)]

- **HF** and **LF** labels
- **universal prediction**
- Data points for different temperatures:
  - 0 nK
  - 40 nK
  - 170 nK

*Figure a*:
- Scatter plot with beta (cm^3/s) on the y-axis and scattering length (a_0) on the x-axis.

*Figure b*:
- Graph with a* (a_0) on the y-axis and \( \eta_\ast \) on the x-axis.
NOT SO UNIVERSAL

[Graph showing data points with annotations: 553.3 G, 554.7 G, 853.1 G, 7.6 G]
NOT SO UNIVERSAL

[Diagram showing data points and error bars for different temperatures and magnetic fields, with annotations for universal prediction, HF, and LF conditions.]
A-D: INDIRECT TESTS

K: INGUSCIO
LI7: KHAYKOVICH, HULET

THEORY: BRAATEN

GOOD AGREEMENT
BUT MODEL DEPENDENT
A-D: INDIRECT TESTS

K: INGUSCIO
LI7: KHAYKOVICH, HULET

THEORY: BRAATEN

GOOD AGREEMENT
BUT MODEL DEPENDENT
A-D: INDIRECT TESTS

K: INGUSCIO
LI7: KHAYKOVI deserialize, HULET

THEORY: BRAATEN

LI6: UEDA, JOCHIM
LI7: KHAYKOVI deserialize

GOOD AGREEMENT BUT MODEL DEPENDENT

TRIMERS BINDING ENERGY

DEVIATIONS AND AGREEMENT
A. Zenesini et al, Non-universal behavior in ultracold atom dimer collisions *In preparation*
HALO DIMER  EFIMOV TRIMER  FOUR-BODY  FIVE BODY

UNIVERSAL  NON-UNIVERSAL

4-2
UNIVERSAL  NON-UNIVERSAL

4-2
CONCLUSIONS

• EFIMOV UNIVERSALITY
  □ FOR DIFFERENT FESHBACH RESONANCES
  □ UP TO FIVE PARTICLES INVOLVED

• NON-UNIVERSALITY
  □ A-D COLLISIONS
LEVT TEAM

R. GRIMM
M. BERNINGER
A. ZENESINI
H.-C. NÄGERL
S. BESLER
B. HUANG
F. FERLAINO
THANKS