Dipolar gases
From dipolar BECs to strongly-correlated lattice gases

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New rules… The dipole-dipole interaction

**Short-range and isotropic**

\[
V(r - r') \approx \frac{4\pi \hbar^2 a}{m} \delta(r - r') \equiv g\delta(r - r')
\]

**External field**

\[
V(\vec{r}) = \frac{d^2}{r^3} \left( 1 - 3\cos^2 \theta \right)
\]

Anisotropic
(partially attractive!)

Long-range interaction

\[
\theta
\]
Dipolar gases: all the way from very weak to huge
Part I: Dipolar BECs

Stability and collapse of dipolar BECs in optical lattices

Roton-like excitations in dipolar BECs
Dipolar BECs

Nonlocal NLSE

\[ i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = \left\{ \begin{array}{c} -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) + g|\psi(\vec{r}, t)|^2 + \\
+ g_d \int d\vec{r}' \frac{(1 - 3\cos^2 \theta)}{|\vec{r} - \vec{r}'|^3} |\psi(\vec{r}', t)|^2 \end{array} \right\} \psi(\vec{r}, t) \]
Instability

The stability of the dipolar BEC depends crucially on the trap geometry

\[
\bar{\epsilon}(q) = \sqrt{\frac{\hbar^2 q^2}{2m} \left[ \frac{\hbar^2 q^2}{2m} + 2m c_s^2(q) \right]} \to \hbar q c_s(q)
\]

Anisotropic sound velocity

\[
c_s(q) \sqrt{g + 4 \frac{\partial}{\partial q} \left( \cos^2 q \right) \frac{1}{3} \frac{1}{m}}
\]

It may be negative! (Phonon instability)

The stability of the dipolar BEC depends crucially on the trap geometry

Collapse


[Lahaye et al., PRL 101, 080401 (2008)]
Non-dipolar gases in lattices

For non-dipolar gases: only on-site interactions

If the hopping is suppressed one has independent (typically low-D) gases at each site
Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important

This is true even if the hopping vanishes (!!!)
A dipolar BEC in an optical lattice

Müller et al., *PRA* 84, 053601 (2011)
Role of inter-site interactions

DDI on-site but cut off inter-site interaction.

Deep lattice regime, hopping is negligible.

Stability strongly affected by inter-site interactions

[Müller et al., PRA 84, 053601 (2011)]
Deconfinement-induced collapse

Interaction energy kept constant

Probe threshold and nature of collapse by change in confinement.
Deconfinement-induced collapse

Procedure:
(i) Prepare at $U_0 \approx 13\ E_R$ and $a \sim 2\ a_0$
(ii) Ramp (100 ms!) to final lattice depth and hold for variable time.
(iii) Perform TOF

→ What is expected?

stable region: nothing happens

unstable region: in-lattice instability

STABLE
$U = 12.6\ E_R$
$a = 2 \pm 2\ a_0$

UNSTABLE
Deconfinement-induced collapse

<table>
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<tr>
<th>$t_{\text{hold}}$ (ms)</th>
<th>$U = 6.3 , E_R$</th>
<th>$U = 3.2 , E_R$</th>
<th>$U = 0 , E_R$</th>
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<td><img src="image14" alt="Image" /></td>
<td><img src="image15" alt="Image" /></td>
</tr>
</tbody>
</table>

Stability threshold: $U = 12.6 \, E_R$, $a = 2 \pm 2 \, a_0$
Deconfinement-induced collapse

Stability threshold

$U = 12.6 \, E_R$

$U = 8.2 \, E_R$

$U = 6.3 \, E_R$

$U = 3.2 \, E_R$

$U = 0 \, E_R$

$\Delta = 2 \pm 2 \, a_0$
Collapse in TOF

Below the threshold:
in-trap collapse → atom number decay

Above the threshold:
no decay → collapse during time of flight
Collapse in TOF

Real time simulations of the nonlocal nonlinear Schrödinger equation including 3-body losses
Collapse in TOF

1. The momentum peaks move very quickly apart from each other ( $t \sim 1/E_R$ )

2. Each individual peak expands very slowly since $\Delta k$ is very small

Coherent

Incoherent
Collapse in TOF

1. The momentum peaks move very quickly apart from each other (t \sim 1/E_R)

2. Each individual peak expands very slowly since \Delta k is very small

3. The central peak acquires the cylindrical geometry of the overall trap. Hence the DDI becomes in average attractive

4. The attractive DDI is not compensated by the very weak kinetic energy associated to \Delta k.

TOF-induced collapse is hence quite peculiar of dipolar lattice gases, since it requires both inter-site coherence and anisotropy of interactions.
Part I: Dipolar BECs

Stability and collapse of dipolar BECs in optical lattices

Roton-like excitations in dipolar BECs
Roton-like excitations

Roton-like minimum in the excitation spectrum (similar to Helium physics)

If the roton energy becomes imaginary, rotons may become unstable whereas phonons are stable: roton instability!!

[O‘Dell et al., PRL 90, 110402 (2003); Santos et al., PRL 90, 250403 (2003); Ronen et al., PRL 98, 030406 (2007); Komineas and Cooper, PRA 75, 023623 (2007)]
Roton-like minimum

It results from the $q$-dependence of the DDI

For pancake traps (in the 3D regime) for large-enough ratio DDI/contact

$$Roton\ minimum\ at\ q_r \sim 1/l_z$$

$$\begin{align*}
(q)^2 & \frac{\hbar^2 q^2}{2m} F \frac{g_d}{g} \frac{\hbar^2 q^2}{2m} + (\hbar z)^2 \\
& = (g + g_d) n_0
\end{align*}$$

Larger density results in a deeper roton

[Santos et al., PRL 90, 250403 (2003)]
Trapped pancake traps: local-density approximation

\[ V(\mathbf{r}) = \frac{m}{2} \left( r^2 + \frac{z^2}{l_z} \right) \]

\[ q \ll \frac{R}{l_z} \ll 1 \]

Local density approximation

\[ (q_R)^2 = \frac{\hbar^2 q^2}{2m} \frac{r^2}{l_z} = \frac{g \gamma}{g} \frac{\hbar^2 q^2}{2m} \left( 1 - \frac{l_z}{R^2} \right) \]

The roton depth is spatially dependent!!
Roton confinement

Minimum in both $q$ and $\rho$

\[
E(\mathbf{q}, \mathbf{r}) = E_0 + \frac{\hbar^2}{2m^*}(\mathbf{q} \cdot \mathbf{q}_r)^2 + \frac{1}{2}m^* \mathbf{r}^2 \]

Roton confinement!

Roton localization length

\[
l^*_\text{rot} = \sqrt{\frac{\hbar}{m^*}} 2^{1/4} \sqrt{\frac{R}{q_r}} \ll R
\]

Lowest roton states

\[
s(\mathbf{q}, \mathbf{r}) \mu \mathbf{e}^{i\mathbf{s}} J_s(q_r) \mathbf{e}^{2/2l^2}
\]

They are crucial in the roton instability

Local roton spectrum will be very important in future experiments trying to prove the roton-like spectrum!
Part I: Dipolar BECs

Stability and collapse of dipolar BECs in optical lattices

Roton-like excitations in dipolar BECs
Part II: Polar lattice gases

Non-polar vs polar gases in optical lattices

Haldane insulator

Interlayer superfluids
Bose-Hubbard Hamiltonian: Mott-insulator phase

\[ H \ N = t \ \left( b_i^+ b_j + H.c. \right) + \frac{U_0}{2} n_i(n_i - 1) n_i \]

Bose-Hubbard Hamiltonian

[Fisher et al., PRB 40, 546 (1989) ; Jaksch et al., PRL 81, 3108 (1998)]

Mott insulator (gapped incompressible insulator)

Superfluid
Bose-Hubbard Hamiltonian: Mott-insulator phase

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Superfluid
Polar bosons in optical lattices

Extended Bose-Hubbard model

\[ H = -t \sum_{i} \left[ b_{i}^{\dagger} b_{i+1} + H.c. \right] + \frac{U_{0}}{2} \sum_{i} n_{i}(n_{i} - 1) + U_{1} \sum_{i} n_{i}n_{i+1} \]

Intersite interactions give a lot of very interesting physics: supersolids, density-wave phases, pair-superfluids, and much more...

Density wave

Supersolid
Part II: Polar lattice gases

Non-polar vs polar gases in optical lattices

Haldane insulator

Interlayer superfluids
Interlayer SF in bilayer polar Fermi gases

[ Pikovski, Klawunn, Shlyapnikov and Santos, PRL 105, 215302 (2010) ]

Cooper-like pairing

(Quasi-)BEC of dimers
Lattice bosons in bilayers: Pair Superfluids

Mott-insulator (MI)

Two-superfluids (2SF)

Pair-superfluid (PSF)

Pair-Supersolid

[Argüelles and Santos, PRA 75, 053613 (2007)]

[Trefzger et al., PRL 103, 035304 (2009)]
Inter-layer filamentation

Bosonic polar molecules in a lattice may form a dipolar chain liquid (DCL) characterized by the BEC of the longest filaments

[Wang, Lukin and Demler, PRL 97, 180413 (2006)]

For fermionic polar molecules chains may be bosonic or fermionic depending on the number of molecules they have!

[Klawunn, Duhme and Santos, PRA 81, 013604 (2010)]
Part II: Polar lattice gases

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Haldane insulator
Extended Bose-Hubbard model

\[ H = -t \sum_i [b_i^+ b_{i+1} + H.c.] + \frac{U_0}{2} \sum_i n_i(n_i - 1) + U_1 \sum_i n_i n_{i+1} \]

Average filling \( \bar{n} = 1 \)

Holstein-Primakoff transformation maps occupation into spin-1

\[ S_i^z = 1 - n_i \]
\[ S_i^+ = \sqrt{2 - n_i} b_i \]
\[ S_i^- = b_i^+ \sqrt{2 - n_i} \]
1D polar bosons in optical lattices at unit filling (spin chain)

Extended Bose-Hubbard model

\[ H = -t \sum_i [b_i^+ b_{i+1}^+ + H.c.] + \frac{U_0}{2} \sum_i n_i (n_i - 1) + U_1 \sum_i n_i n_{i+1} \]

The system resembles to a large extent an AF spin-1 chain with uniaxial single-ion anisotropy

\[ H = J \sum_i \left[ S_i^x \cdot S_{i+1}^x + S_i^y \cdot S_{i+1}^y + \Delta S_i^z \cdot S_{i+1}^z + D (S_i^z)^2 \right] \]

\[ J = 2t \quad D = U_0 / 4t \]

\[ \Delta = U_1 / 2t \]

(imperfect mapping: due to \( n > 2 \) and extra terms \( \sim S_i^- \cdot F(S_i^z + S_{i+1}^z) \cdot S_{i+1}^+ \))
AF spin-1 chains

\[ H = J \sum_i \left[ S_i^x \cdot S_{i+1}^x + S_i^y \cdot S_{i+1}^y + \Delta S_i^z \cdot S_{i+1}^z + D \left( S_i^z \right)^2 \right] \]

Haldane phase
(„diluted AF order“)

String order: \[ \lim_{|i-j| \to \infty} \left\langle -S_i^z \exp \left[ i\pi \sum_{l=i+1}^{j-1} S_l^z \right] S_j^z \right\rangle \neq 0 \]

Large-D phase

Parity order: \[ \lim_{|i-j| \to \infty} \left\langle \exp \left[ i\pi \sum_{l=i+1}^{j-1} S_l^z \right] \right\rangle \neq 0 \]

Néel phase

[Chen, Hida and Sanctuary, PRB 67, 104401 (2003)]
1D polar gases in optical lattices: Haldane-insulator phase

[Dalla-Torre, Berg and Altman, PRL 97, 260401 (2006)]

\[ H = -t \sum_i \left[ b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2} \sum_i n_i (n_i - 1) + U_1 \sum_i n_i n_{i+1} \]

Haldane-insulator

[Dalla Torre et al., PRL 97, 260401 (2006)]

String order: \( \mathcal{O}_s^O \equiv \lim_{|i-j| \to \infty} \left\langle n_i \exp \left[ i \sum_{l=i+1}^{j-1} n_l \right] \right\rangle \neq 0 \)

Mott-insulator

Parity order: \( \mathcal{O}_p^O \equiv \lim_{|i-j| \to \infty} \left\langle \exp \left[ i \sum_{l=i+1}^{j-1} n_l \right] \right\rangle \neq 0 \)

Experiment [Endres et al., Science 334, 200 (2011)]

Density wave

\[ \ldots 101...121...101...121... \]

\[ \Delta \]

\[ U_0/t \]

\[ \ldots 1201...1201...1201... \]

\[ \ldots 0202020202020202 \ldots \]
1D polar bosons in optical lattices

Extended Bose-Hubbard model

\[ H = \sum_i t b_i^+ b_{i+1} + H.c. + \frac{U_0}{2} \sum_i n_i(n_i-1) + \sum_{i<j} U_{ij}n_in_j \]

The intersite interactions lead to a very rich physics for 1D chains and ladders

- **Supersolid** [Batrouni et al., PRL 97, 087209 (2006)]
- **Devil’s staircase** [Burnell et al., PRB 80, 174519 (2009)]
- **Haldane insulator** [Dalla Torre et al., PRL 97, 260401 (2006)]
- **Disorder** [Xeng et al., arXiv (2012)]
- **Simulation of spin-orbital models** [Sun et al., PRB (2012)]
- **More... see** [Baranov et al, arXiv:1207.1914]
Part II: Polar lattice gases

Non-polar vs polar gases in optical lattices

Haldane insulator

Interlayer superfluids
A. Argüelles, M. Klawunn, R. Nath, M. Jona-Lasinio, A. Pikovski
X. Deng, K. Łakomy, F. Wächtler

T. Vekua (ITP)
M. Fattori, G. Modugno and their group
T. Pfau, A. Griesmaier and their group
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