Multi-color modulation on a 1D optical lattice

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Outline:

1. Lattice shaking
2. Lattice depth modulation
3. Triple modulation
The Bose-Hubbard Model

\[ \hat{H} = -J \sum_i (\hat{b}_i \hat{b}_i^\dagger + \hat{b}_i^\dagger \hat{b}_{i+1}) + U \sum_i \hat{n}_i (\hat{n}_i - 1) \]

J ~ 10 to 100 Hz  U ~ 100 to 1000 Hz  V ~ 10 KHz
1. Lattice shaking
Tunneling control

\[
\hat{H} = \hat{H}_{BH} + K \cos(\omega t) \sum_j j \hat{n}_j \\
J_{\text{eff}} = J J_0 \left( \frac{K}{\hbar \omega} \right)
\]

\(J_0\): 0th Bessel function, first \(J\) suppression at \(K/\hbar \omega = 2.4\) ⇒ Heating!!

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Lignier et al, PRL 2007
Tuneable gauge potential

Periodic shakings, not sinusoidal, induces a phase on the hopping.

\[ \hat{H} = -J_{\text{eff}} \sum_i (e^{i\phi} \hat{b}_i \hat{b}^\dagger_{i+1} + h.c.) + \hat{U} \]
Ferromagnetic domains

Hybridizing the lowest Bloch band by shaking the lattice at a frequency slightly blue-detuned from the first band gap.

$\lambda = 1064\ \text{nm} \quad \Delta x = 65\ \text{nm}$

Weak shaking, no heating.

Hybridizing the lowest Bloch band by shaking the lattice at a frequency slightly blue-detuned from the first band gap.

Parker et al, Nat. Phys. 2013
Meissner to vortex superfluid

Raman induced hopping on the ladder rungs.

Transition driven by the ratio $K$ (rung hopping) over $J$ (leg).

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Atala et al, Nat. Phys. 2014
2. Lattice depth modulation
Tilted lattice

$\hat{H} = \hat{H}_{BH} + \Delta \sum_j j \hat{n}_j$

$\Delta = \text{lattice tilting, realizable by means of the gravitational potential.}$

Due to the tilting the system is nearly frozen, no hopping in there!

$J = 30 \text{ Hz} \quad U = 300 \text{ Hz}$
$\Delta = 900 \text{ Hz} \quad V = 10 \text{ KHz}$
Depth modulation of a tilted lattice in the linear regime

\[ \hat{H} = \hat{H}_{BH} + \Delta \sum_j j \hat{n}_j + \delta J \cos(\omega t) \sum_j (b_{j+1}^\dagger b_j + h.c.) \]

in the linear regime

\[ \downarrow \]

periodic modulation of \( V(t) \) = periodic modulation of \( \delta J(t) \).

\[ \delta V \approx 10/15\% V \]
J = 30 Hz     U = 300 Hz
\( \Delta = 900 \text{ Hz} \quad V = 10 \text{ KHz} \]

Ma et al, PRL 2011

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Depth modulation of a tilted lattice

hopping processes occur at different energies

\[ \downarrow \]

scan over the frequencies

\[ \downarrow \]

resonance with a hopping process

\[ \downarrow \]

controllable assisted hopping

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3. Triple modulation
### Three selected hopping processes

<table>
<thead>
<tr>
<th>Process</th>
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<th>Mod. frequency</th>
<th>Phase</th>
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<tr>
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<td>$\Delta$</td>
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<tr>
<td>b(i)</td>
<td>$\Delta + U$</td>
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<tr>
<td>b(ii)</td>
<td>$\Delta - U$</td>
<td>$\Delta - U + \tilde{U}$</td>
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\[
\hat{H} = \hat{H}_{BH} + \sum_i \delta J_i \cos(\omega_i t + \phi_i) \sum_j (\hat{b}_{j+1}^\dagger \hat{b}_j + h.c.)
\]

\[
V(t) = V_0 + \sum_{i=1}^{3} \delta V_i \cos(\omega_i t + \phi_i)
\]

Each depth modulation is individually mapped into a resonant hopping term.

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Effective Hamiltonian

\[ H_{\text{eff}} = -\frac{\delta J}{2} \sum_{\sigma, <l,k>} c_{l,\sigma}^\dagger e^{i\phi} |n_{\sigma,l} - n_{\sigma,k}| c_{k,\sigma} + \tilde{U} \sum_{j,\sigma,\sigma'} n_{\sigma,j}n_{\sigma',j} + J^2 \sum_{k \neq n} \frac{\hat{P}_{kn}}{E^{(0)}_n - E^{(0)}_k} \]

density-dependent Peierls phase

effective on-site interaction

2° order correction

We focus on the case in which two lasers are slightly detuned (red/blue) of the same frequency from their resonance and dephased of \( \phi \) from the third one.

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Second-order corrections

\[ J^2 \sum_{k \neq n} \frac{\hat{P}_{kn}}{E_n^{(0)} - E_k^{(0)}} \]

In the dynamics, take them into account!
Tuneable on-site interaction

\[ \tilde{U} \sum_{j,\sigma,\sigma'} n_{\sigma,j} n_{\sigma',j} \]

I. Easy to tune, it corresponds to the detuning of the laser.

II. It can be controlled independently from the bare/resonant hopping.

III. It allows to control the interaction in atoms devoid of Feshbach resonance (Ytterbium).
Density-dependent gauge

\[-\frac{\delta J}{2} \sum_{\sigma, <l,k>} c_{l,\sigma}^\dagger e^{i\phi|n_{\sigma} - n_{\bar{\sigma}},k|} c_{k,\sigma}\]

I. The induced phase is density-dependent.

II. The density dependence is on the other fermionic components.

III. Space-dependent vector potential, in synthetic dimension, the kick

IV. One could study how the Meissner to vortex transition changes according to random density distrib, and maybe adapts automatically to one or another
Summary & Outlook

Density-dependence, Anyon model, Density-dependent magnetic field, quantum Hall physics.

No heating, U tuneable too.

Phase tuning.