Coherence time of a quantum gas

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Plan

1. Time coherence of a BEC
   - Motivation
   - Considered System and observables
   - Previous results
2. Bosons
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   - Analytical results
   - Analytics and interpretation
   - How to measure and numbers
3. Fermions
   - Interest and difficulties
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   - Example : the unitary Fermi gas

How long is the intrinsic coherence time of BEC ?

Fundamental interest :

Coherence is a key property of Bose-Einstein condensates

- Spatial coherence extends over the entire system size.
- What about time coherence ?

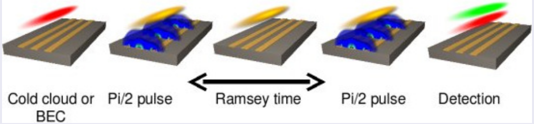
Trapped cold gas = isolated system

Intrinsic coherence time of an isolated "macroscpic" quantum system

Interest for applications

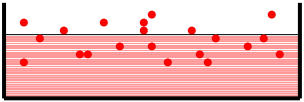
Interferometric measurements (internal or external states)

Clocks with trapped atoms, e.g. on an Atom chip



Considered System

We consider : 3D system, finite size, N fixed, $T < T_c$, condensate made of bosons or paires of opposite spin fermions



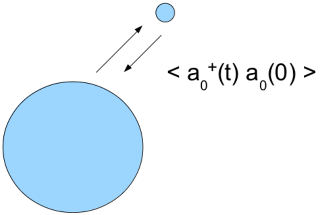
Time coherence is described by the time correlation function

$$g_1(t,0) = \langle \hat{\psi}^\dagger(\mathbf{r},t)\hat{\psi}(\mathbf{r},0)\rangle \overset{t\to\infty}{\sim} |\phi_0(\mathbf{r})|^2 \langle \hat{\mathbf{a}}_0^\dagger(\mathbf{t})\hat{\mathbf{a}}_0(\mathbf{0})\rangle$$

- $T = 0$: **The correlation function oscillates (no decay)** at a frequency μ_0/h **Beliaev '60s**
- $T \neq 0$: **The correlation function decays** thermal excitations = dephasing environment

Observables of interest

Time correlation function of condensate amplitude $\langle a_0^\dagger(t) a_0(0) \rangle$ **:**
Accessible by an interferometric measurement.



Modulus-phase representation : $a_0 = e^{i\theta}\sqrt{N_0} \quad , \quad [N_0, \theta] = i$

For a Gaussian probability distribution of $\theta(t) - \theta(0)$ and weak fluctuations of N_0

$$|\langle a_0^\dagger(t) a_0(0) \rangle| \simeq \langle N_0 \rangle \exp\left\{ -\frac{1}{2} \mathrm{Var}\, [\theta(\mathbf{t}) - \theta(\mathbf{0})] \right\}$$

The correlation function decays as the variance of the accumulated phase grows $\mathrm{Var}\, [\theta(t) - \theta(0)]$ **as a function of time** t.

Previous predictions on phase spreading at $T \neq 0$

Bosons

- Quantum optics inspired approaches : Treat the non condensed particles as a bath R. Graham, C. Gardiner, P. Zoller '98-2000

  The phase spreading is **DIFFUSIVE** : $\mathrm{Var}\,[\theta(t) - \theta(0)] \sim Dt$

- Many-body approach (but interactions among Bogoliubov modes were neglected) A. Kuklov, J. Birman '98

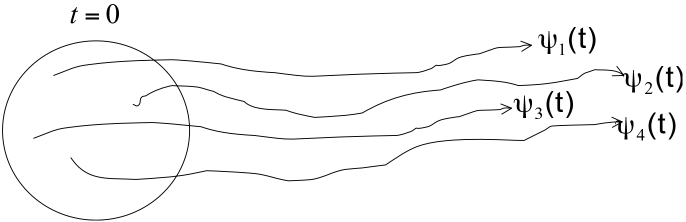
  The phase spreading is **BALLISTIC** : $\mathrm{Var}\,[\theta(t) - \theta(0)] \sim \alpha t^2$

Paired Fermions : no predictions.

Bosons : Classical Field simulations

ψ describes condensate and non-condensed fraction

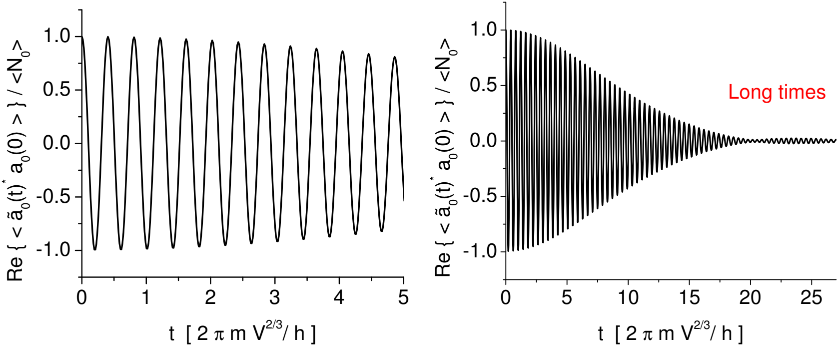
$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\Delta}{2m} + g|\psi(\vec{r},t)|^2\right]\psi$$



1. Stochastic fields $\psi_i(0)$ sampling $\sigma = \frac{1}{Z}e^{-\beta E[\psi,\psi^*]}$ (canonical ensemble)
2. Deterministic evolution with NLSE
3. Calculate the averages at time t.

Amplitude correlation function

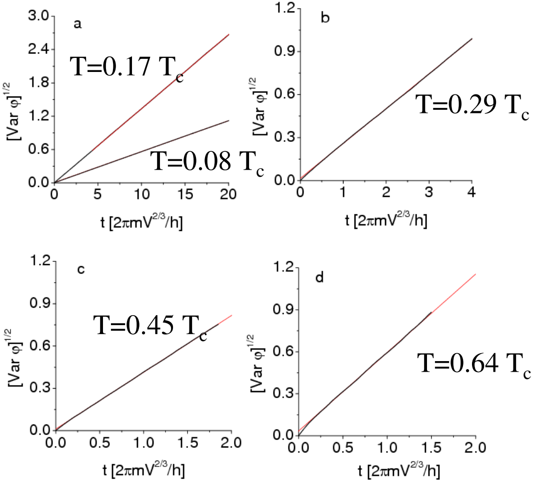
$N = 4 \times 10^5, \; T = 0.17\, T_c, \; \mu = 700, \; T_C = 18000 \; (\hbar^2/mV^{2/3}).$



1) **The correlation function decays** $\langle a_0^*(t) a_0(0) \rangle \to 0$ **for** $t \to \infty$

Trivial factor $\exp(i\rho g t/\hbar)$ **eliminated**

Variance of the accumulated phase $\mathrm{Var}\,[\theta(t) - \theta(0)]$

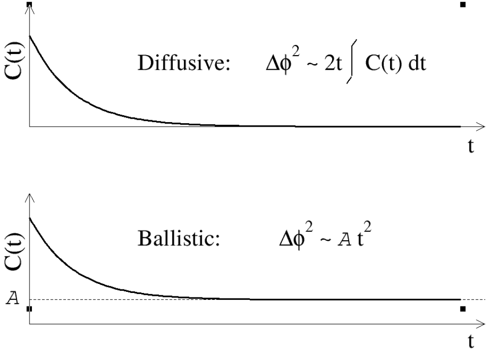


2) Phase difference $\theta(t) - \theta(0)$ spreads ballistically over a wide range of temperatures.

Correlation function of $\dot{\theta}(t)$ and phase variance

Correlation function : $C(t) = \langle \dot{\theta}(t)\dot{\theta}(0)\rangle - \langle \dot{\theta} \rangle^2$

Phase variance : $\mathrm{Var}\,[\theta(\mathbf{t}) - \theta(\mathbf{0})] = 2t \int_0^t C(\tau)d\tau - 2\int_0^t \tau C(\tau)d\tau$

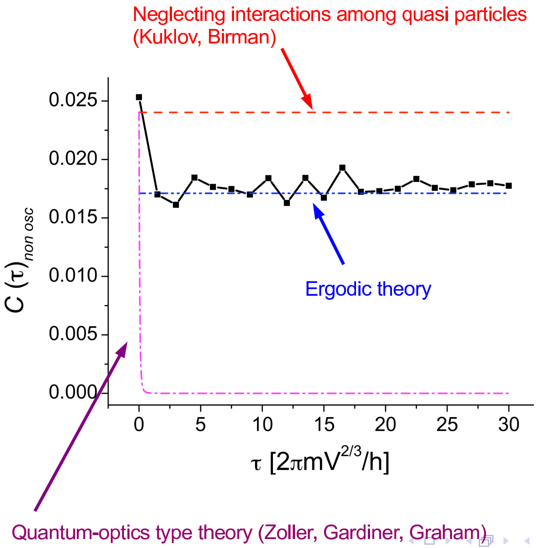


The longtime behavior of the phase depends on the limit $\mathcal{A}$ of $C(t)$

- $\Rightarrow \mathcal{A} = 0 \Rightarrow$ **diffusive**
- $\Rightarrow \mathcal{A} \neq 0 \Rightarrow$ **ballistic**

Illustration by a classical field simulation

Correlation of the condensate phase derivative $C(\tau) = \langle\dot\theta(\tau)\dot\theta(0)\rangle - \langle\dot\theta\rangle^2$



Role of the Bogoliubov occupation numbers n_k

Condensate phase derivative $\dot{\theta}(t) = \sum_k A_k n_n$

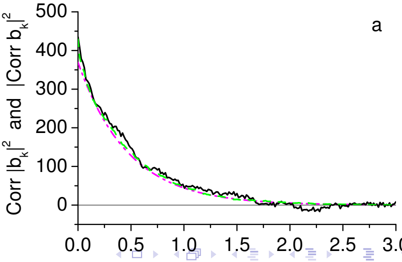
Bogoliubov : $\langle n_k(t) n_{k'}(0)\rangle - \langle n_k\rangle\langle n_{k'}\rangle = \delta_{k,k'}\, \langle n_k\rangle^2$

Gaussian model or ME (Landau and Beliaev damping) :
The b_k are Gaussian variables with a finite correlation time

$$\langle n_k(t) n_{k'}(0)\rangle - \langle n_k\rangle\langle n_{k'}\rangle = \delta_{k,k'}\, \langle n_k\rangle^2\, e^{-\Gamma_k t}$$

Ergodic theory :
The $b_k(t \to \infty)$ uniformly explore a fixed-energy surface

$$\langle n_k(t \to \infty) n_{k'}(0)\rangle - \langle n_k\rangle\langle n_{k'}\rangle \neq 0$$



Predictions of the ergodic model

- For a system prepared in the **canonical ensemble**, the correlation function of the phase derivative does not vanish at long times. (Different from quantum-optics like models).

$$C(t) = \langle \dot{\theta}(t)\dot{\theta}(0)\rangle - \langle \dot{\theta}\rangle^2 \to \mathcal{A} \neq 0 \quad \text{for} \quad t \to \infty$$

- The condensate **phase spreads ballistically**

$$\mathrm{Var}\,[\theta(t) - \theta(0)] \sim \mathcal{A}t^2$$

- The coefficient $\mathcal{A}$ is given by the ergodic theory :

$$\mathcal{A} = \frac{1}{\hbar^2}\left(\frac{d\langle\mu\rangle_{\mathrm{mc}}}{dE}\right)^2_{E=\bar{E}} \mathbf{Var}\ H$$

- **Ergodicity is ensured by the interactions among Bogoliubov quasiparticles.**
- **The ergodic model can be generalized to the quantum case**

Ergodic theory (1)

The long time limit of the correlation function $C(t) = \langle \dot\theta(t)\dot\theta(0)\rangle - \langle\dot\theta\rangle^2$ can be found with an ergodic theory

- **Replace the long time limit with the time average**

$$C_A(+\infty) \equiv \lim_{t\to+\infty} \frac{1}{t}\int_0^t d\tau \, \left[\langle A(\tau)A(0)\rangle - \langle A\rangle^2\right].$$

- **Use the exact eigenstates** : $\rho = \sum_m p_m |m\rangle\langle m|$

  In the absence of degeneracies you get:

$$C_A(+\infty) = \sum_m p_m \langle m|A|m\rangle^2 - \left[\sum_m p_m \langle m|A|m\rangle\right]^2 .$$

- **"Eigenstate thermalization hypothesis" The average of an observable A on an eigenstate $|m\rangle$ is close to the microcanonical average**

$$\langle m|A|m\rangle \simeq \langle A\rangle_{\rm mc}(E_m)$$

J. M. Deutsch PRA (1991), M. Rigol, PRL (2009).

Ergodic theory (2)

Applied to our case : correlation function of the phase derivative $A = -\hbar\dot{\theta} \equiv \mu$

$$C_{\dot{\theta}}(+\infty) = \sum_E p(E)\ \left[\langle\mu\rangle_{\rm mc}(E) - \langle\mu\rangle_{\rm mc}(\bar{E})\right]^2 = \left(\frac{d\langle\mu\rangle_{\rm mc}}{dE}\right)^2_{E=\bar{E}} \textbf{Var } H$$

Consequence on phase spreading:

$$\mathrm{Var}\left[\theta(t) - \theta(0)\right] \sim \frac{t^2}{\hbar^2}\left(\frac{d\langle\mu\rangle_{\rm mc}}{dE}\right)^2_{E=\bar{E}} \textbf{Var } H$$

Physical interpretation:

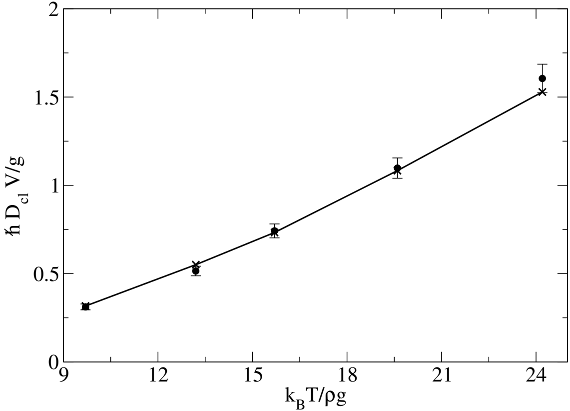
For a given realization of energy E, at long times

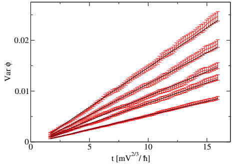
$$\theta(t) - \theta(0) \sim -\frac{\mu(E)t}{\hbar} \qquad \Delta\left[\theta(t) - \theta(0)\right] \sim \frac{t}{\hbar}\ \frac{d\mu}{dE}\bigg|_{E=\bar{E}} \Delta E$$

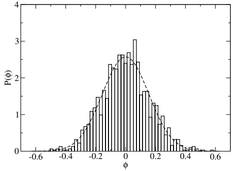
What happens in the micro canonical ensemble when $C_{\dot\theta}(+\infty) = 0$?

$\mathrm{Var}[\theta(t) - \theta(0)] = Dt$ **phase diffusion** $\hbar\,\mathbf{D}\,V/g = f\,(\,k_B T/\rho g\,)$

$-x-$ **Classical Kinetic equations** • **Classical Field Simulation**







Quantitative test: classical kinetic theory on a lattice (with a cut-off)

Summary of results (weakly interacting bosons)

- Using ergodic theory we predict the long time behavior of the phase for a finite temperature condensate:

$$\mathrm{Var}[\theta(t) - \theta(0)] \simeq \mathcal{A}t^2 \quad \text{with} \quad \mathcal{A} \propto \mathrm{Var}\, E$$

- To describe the approach to the long time limit, we write kinetic equations and explicitly calculate

$$\mathrm{Var}[\theta(t) - \theta(0)] \simeq \mathcal{A}t^2 + \mathcal{B}t + \mathcal{C} \quad \text{for} \quad t \to \infty$$

  The coefficient $\mathcal{B}$ (phase diffusion) sets the limit of phase coherence in a finite temperature BEC.

  A. Sinatra, Y. Castin, E. Witkowska, PRA (3 papers 2006-2009) "New Trends and Hot Topics in Atomic and Polariton Condensates" (2013) book chapter

- Collaboration with P. Treutlein for the first experimental determination of the BEC coherence time.

How to measure the time correlation function

Initially all atoms are in state a
Ramsey scheme with very small angle pulse $\eta \ll 1$.
The first pulse is at time 0, the second pulse is at time t.

$$\hat{\psi}_a(r, 0^+) = \sqrt{1-\eta^2}\,\hat{\psi}_a(r, 0^-) + \eta\hat{\psi}_b(r, 0^-)$$

$$\hat{\psi}_b(r, 0^+) = \sqrt{1-\eta^2}\,\hat{\psi}_b(r, 0^-) - \eta\hat{\psi}_a(r, 0^-)$$

- No interactions a-b (reduce the spatial overlap)
- No interactions b-b (very dilute cloud)

$\hat{b}_0(t^-) = e^{i\delta t}\hat{b}_0(0^+)$ $\quad\quad$ $\delta = \omega_L - \omega_{ab}$ $\quad\quad$ rotating frame at frequency ω_L

After the second pulse measure : $\langle \hat{b}_0^\dagger(t^+)\hat{b}_0(t^+)\rangle$

How to measure the time correlation function

For $\eta \ll 1$:

Interference fringes in the population of b atoms

$$N_{b0}(t) = 2\eta^2 \left\{ \langle \hat{n}_0 \rangle + \mathrm{Re}\left[e^{i\delta t} \langle \hat{a}_0^\dagger(t)\hat{a}_0(0) \rangle \right] \right\} + O(\eta^4)$$

- The signal is small: one must **detect a small number of atoms**
- The **contrast is independent of η** : equal to 1 at $t = 0$

By varying $\delta t \equiv (\omega_L - \omega_{\mathrm{ab}})t$ $\qquad \frac{N_{b0}^{\max} - N_{b0}^{\min}}{N_{b0}^{\max} + N_{b0}^{\min}} = \frac{|\langle \hat{a}_0^\dagger(t)\hat{a}_0(0)\rangle|}{\langle \hat{a}_0^\dagger(t)\hat{a}_0(t)\rangle} =$ **contrast**

Example of Numbers for the ballistic phase spreading

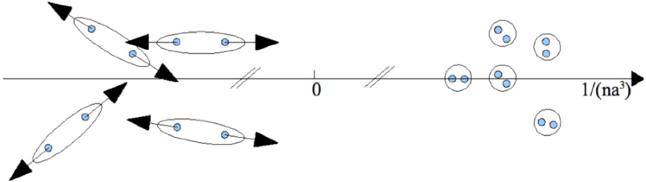
$N = 4 \times 10^5$, $k_B T/\rho g = 3$, $a = 5.3\mathrm{nm}$, $T = 0.3 T_c = 0.14\mu\mathrm{K} \Rightarrow$
$t_{\mathrm{br}} = 750\mathrm{ms}$

Blurring times of tens of ms are predicted in small samples.

Phase dynamics of a condensate of paired fermions

Interest of fermionic gases

Possibility to explore different interaction regimes $-\infty < \frac{1}{na^3} < +\infty$



$\rightarrow$ **Introduce the condensate phase operator and study its dynamics at $T \neq 0$**

- **No Bogoliubov theory available**
- **Two excitation branches F,B**
- **Interactions between excitations**

General theory for phase coherence at $T \neq 0$ (fermions and bosons)

K.Kurkjian, Y.Castin, A.Sinatra, arXiv:1502.05644

Phase operator and its time derivative

The condensate wave function $\phi(\mathbf{r}_1, \mathbf{r}_2)$ is the eigenvector of the 2-body density matrix $\rho_2(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \langle \hat{\psi}^\dagger_\uparrow(\mathbf{r}'_1)\hat{\psi}^\dagger_\downarrow(\mathbf{r}'_2)\hat{\psi}_\downarrow(\mathbf{r}_2)\hat{\psi}_\uparrow(\mathbf{r}_1)\rangle$ with macroscopic eigenvalue $\bar{N}_0$

The Condensate phase $\hat{\theta}_0$ is the phase of the operator $\hat{a}_0$ obtained by projecting the pairing field $\hat{\psi}_\downarrow(\mathbf{r}_2)\hat{\psi}_\uparrow(\mathbf{r}_1)$ onto $\phi(\mathbf{r}_1, \mathbf{r}_2)$.

The decay of the pair correlation function is then given by

$$g_1(t) = \langle \hat{a}_0^\dagger(t)\hat{a}_0(0)\rangle \simeq \bar{N}_0 \langle e^{-i\hat{\theta}_0(t)} e^{i\hat{\theta}_0(0)}\rangle$$

Coarse grained average of phase derivative

$$-\frac{\hbar}{2}\overline{\frac{d\hat{\theta}_0}{dt}}^t = \mu_0(\hat{N}) + \sum_{s=F,B}\sum_\alpha \frac{d\epsilon_{s,\alpha}}{dN}\hat{n}_{s,\alpha}$$

First microscopic derivation at the operator/multimode level
Using a generalized coherent state Ansatz including moving pairs (Blaizot-Ripka)

Decay of the time correlation function

The system state : $\hat{\rho} = \sum_\lambda \Pi_\lambda |\psi_\lambda\rangle\langle\psi_\lambda|$ **with** $\hat{H}|\psi_\lambda\rangle = E_\lambda|\psi_\lambda\rangle$

Correlation function in a many-body eigenstate ψ_λ

$$g_1^\lambda(t) \simeq \bar{N}_0 e^{iE_\lambda t/\hbar}\langle\psi_\lambda|e^{-i(\hat{H}+\hat{W})t/\hbar}|\psi_\lambda\rangle \qquad \hat{W} = \hbar\frac{d\hat{\theta}_0}{dt} + O(\frac{1}{N})$$

$$\simeq \bar{N}_0\, \mathrm{e}^{-\mathbf{it}\langle\psi_\lambda|\hat{\mathbf{W}}|\psi_\lambda\rangle/\hbar}\, \mathrm{e}^{-(\mathrm{i}\delta_\lambda+\gamma_\lambda)\mathbf{t}}$$

Leading term for the decay of $g_1^\lambda(t)$

$$\langle\psi_\lambda|\frac{d\hat{\theta}_0}{dt}|\psi_\lambda\rangle \stackrel{ETH}{=} \mu_{\mathrm{mc}}(N_\lambda, E_\lambda)$$

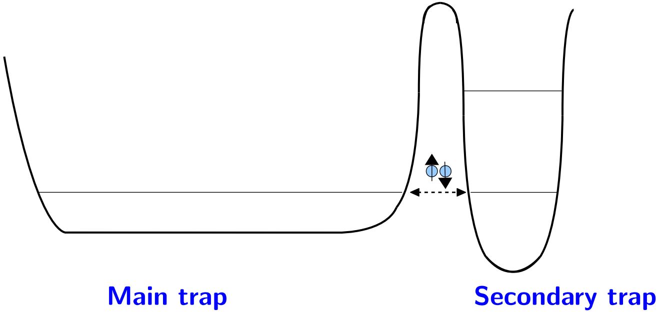
Gaussian decay of g_1 after ensemble average (ballistic phase spreading) $t_{\mathrm{br}} \propto N^{1/2}$.

Subleading term quadratic in $\hat{W}$

$$\gamma_\lambda + i\delta_\lambda = \int_0^{+\infty} dt \left\{ \left\langle \frac{d\hat{\theta}_0(t)}{dt}\frac{d\hat{\theta}_0(0)}{dt}\right\rangle_\lambda - \left\langle\frac{d\hat{\theta}_0}{dt}\right\rangle_\lambda^2 \right\}$$

Exponential decay of g_1 even in the microcanonical ensemble (phase diffusion) $t_{\mathrm{diff}} \propto N$

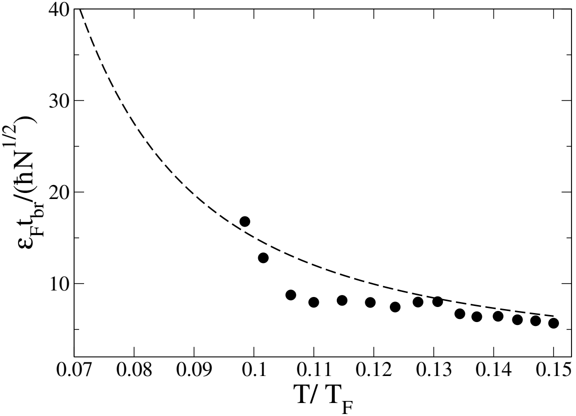
Proposed experimental configuration and protocol



- Ramsey interferometry to measure $g_1(t)$: two pulses of small angle separated by time t
- **Dimerize the pairs for preparation, pluses and detection**
- $\langle n_{\rm sec}\rangle$ oscillates at $\omega = 2(\mu_{\rm main} - \mu_{\rm sec})/\hbar$, the contrast is $|g_1(t)/g_1(0)|$

Thermal blurring time of a unitary Fermi gas $(a = \infty)$

From equation of state measurements (Salomon LKB, Zwierlein MIT)



Canonical ensemble $T = 0.12\,T_F \simeq 0.7\,T_c,\ N = 10^5,\ T_F = 1\mu K$

Then : $t_{\rm br} = 20\text{ms}$

Conclusions of BEC coherence time (Bose/Fermi)

- In a finite size condensed system at non-zero temperature, the time correlation function decays even when the system is isolated in its evolution and the particle number N is fixed.
- The loss of phase memory is due to interactions of the condensate with the excited modes that constitute a dephasing environment.
- $\hat{\theta}_0$ evolves at long times as $-2\mu_{\mathrm{mc}}(E)t/\hbar$ where $\mu_{\mathrm{mc}}(E)$ is the microcanonical chemical potential; energy fluctuations from one realization to the other then lead to a ballistic spreading of the phase and to a Gaussian decay of the temporal coherence function with a characteristic time $\propto N^{1/2}$.
- In the absence of energy fluctuations, the coherence time scales as N due to the diffusive motion of $\hat{\theta}_0$.
- We predict a coherence time of tens of milliseconds for the canonical ensemble unitary Fermi gas.